Estimating the source parameters of a moderate earthquake using the second seismic moments

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Introduction

The study of earthquake generation and associated seismic parameters such as seismic moment, rupture size, rupture velocity and direction, and stress drop is crucial for understanding earthquake dynamics and the underlying physics of the seismic process. This information plays an important role in the estimation of ground shaking near the earthquake source and in the assessment of seismic hazard, even for low to moderate magnitude earthquakes.

The kinematic properties of small earthquakes are often difficult to determine, and simple models are often used to represent these events, although improved records show that source complexity is common even for small earthquake ruptures (e.g. Calderoni and Abercrombie, 2023 and reference therein).

A critical task in determining finite source attributes for moderate and low magnitude earthquakes requires good removal of path and site effects. To address this problem, several methods based on empirical Green's function (EGF) deconvolution have been developed in recent decades. Although the EGF offers several advantages, its application is associated with some difficulties, as there are often no focal mechanisms for small earthquakes and source effects have been observed even for low energy events (Calderoni et al. 2023).

The simplest general representation of an earthquake that contains information about the rupture extent and directivity is the point-source representation plus the variances or second-degree moments of the moment-release distribution. The hypocenter and the origin time of the earthquake correspond to the spatial and temporal average (first degree moment) of the release moment distribution. The information about the rupture extent, the characteristic duration and the direction of rupture propagation correspond to the variance of the moment distribution in the spatial, temporal and spatio-temporal domain (second-degree moments). Seismic moments are calculated from apparent durations measured from apparent source time functions (ASTF) for each station after removal of path effects. The ASTF is thus the projection of the rupture process onto the seismic ray path, and its properties also depend on the azimuth and take-off angles (e.g. McGuire, 2004). For a unilateral rupture, the ASTF observed from stations in the direction of propagation would be significantly shorter than the ASTF from stations in the opposite direction.

A major advantage of the second moments method is that it can theoretically be applied to all earthquakes, regardless of their magnitude and complexity, and without requiring the assumptions of an a priori source model (e.g. McGuire 2004; Meng et al., 2020; Cuius et al., 2023). It is also a consistent tool for evaluating scaling relationships between finite source attributes and earthquake magnitudes for large and small earthquakes and for resolving fault plane ambiguity.

However, the elimination of the path effect is crucial, and a biased ASTF calculation would lead to inaccurate calculations of the second seismic moments. However, there may also be other factors that influence the results of the second moments, even if the propagation effects have been correctly removed.

The aim of this study is to implement and test an efficient method for estimating source parameters and rupture directivity in near real-time for medium and small earthquakes. To achieve our goal, we implemented an approach developed by McGuire et al. (2004), which consists of calculating the second-degree seismic moments (Meng et al., 2020; Cuius et al., 2023). In this paper, we first perform a study with some synthetic tests to evaluate the influence of uncertainties related to our prior knowledge and observations on the resulting source parameters (Cuius et al. 2023). We then apply the method to a real earthquake in Italy and present the result.

Analysis of the sensitivity of the second moments tensor resolutions

To evaluate the sensitivity of the second moment solutions, we used synthetic ASTFs computed for a rectangular plane fault discretized by a grid of cells, each assigned a specific slip value. Full details can be found in Cuius et al. 2023. The input parameters used to model the ASTF for a magnitude Mw 4.6 earthquake source are listed in Tab. 1. We assumed that the epicenter was located in central Italy and approximated the fault as a 3.0 km box model (Fig. 1). The rupture area was divided into 12x12 cells, and the slip distribution and rupture time for the unilateral (Fig. 1a; 1b) and bilateral (Fig. 1d; 1e) scenarios were taken from a previous study of a similar magnitude earthquake (SRCMOD database - Mai and Thinbgaijam, 2014), with a focal mechanism of 247° strike, 46° dip and 40° dip. Using the actual station configuration, we calculated the ASTFs with a sampling frequency of 100 Hz and a source time function of 3 seconds. A uniform propagation of the rupture front with a rupture velocity of 2.75 km/s was assumed, which corresponds to 0.9 times the S-wave velocity in the source region. A simplified 1-D velocity model for central Italy was used to model the ASTF (Cuius et al., 2023).

<table>
<tr><th></th><th colspan="5">Unilateral rupture</th><th colspan="5">Bilateral rupture</th></tr>
<tr><td></td><td>(km)</td><td>(km)</td><td>(sec)</td><td>| (km/s)</td><td>Dir</td><td>(km)</td><td>(km)</td><td>(sec)</td><td>| (km/s)</td><td>Dir</td></tr>
<tr><td>Input parameters</td><td>1.39</td><td>1.21</td><td>0.42</td><td>2.64</td><td>0.80</td><td>1.39</td><td>1.21</td><td>0.31</td><td>1.13</td><td>0.25</td></tr>
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Tab. 1. Input parameters used to model the unilateral and bilateral scenarios for the characteristic rupture size (and), characteristic rupture duration ( ), centroid rupture velocity ( ) and directivity (dir).

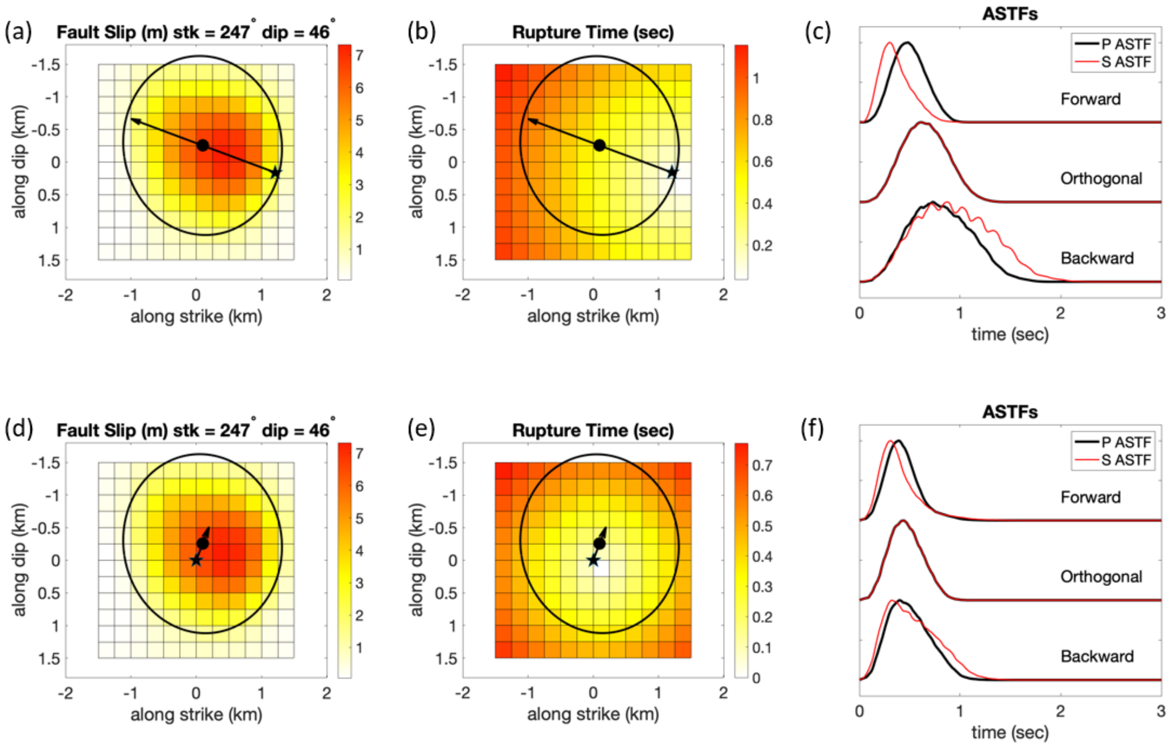

Fig 1. Input source for unilateral (A,B) and bilateral (D,E) scenarios. The star represents the hypocenter, the dot represents the centroid location, and the arrow indicates the rupture direction. Panels (C,F) show the ASTFs calculated from the respective models for three different azimuth directions.

To investigate how the uncertainties introduced by the input data may affect the solutions of the resolved second seismic moments, we used the bootstrap approach. In this technique, perturbations are introduced for each input parameter to be analyzed by generating 1000 variations around the mean value. An inversion is then performed to assess the impact on the mean and standard deviation of the resulting data. The workflow is summarized in Fig. 2.

We investigated the uncertainties associated with the ASTF, the location of the hypocenter, the station distributions around the source, the focal mechanism, and the velocity model used for ray tracing. Some of these tests are interrelated. For example, the uncertainties in the position of the hypocenter and the velocity model affect the calculated ray path, and both the different focal mechanism and station coverage affect the resolution of the fault plane. The uncertainties in the epicenter estimates were not investigated because they have negligible effects on the slowness vectors in the inversion of the second moments.

Results of the synthetic tests

The sensitivity analysis performed in this study shows that the uncertainties in the input data have different effects on the calculation of the source parameters and that an accurate measurement of the ASTF as well as the velocity model play the most important role in influencing the inversion process. The results of our tests (Tab. 2 and Fig. 3) show that the main source parameters, i.e. fracture size, swelling duration and centroid velocity, are generally well reproduced within the standard deviation. The source duration resulting from the inversion process is strongly influenced by the duration of the input ASTF, and even 10 % influences the inversion of the second moment tensor. In the case of dense instrumentation, the horizontal location of the earthquake can be well resolved, but the resolution of the earthquake depth is largely determined by the velocity model, and an inaccurate earthquake location can lead to uncertainties in the resolved second moments. Care must also be taken to avoid artifacts due to the discretization of the velocity model when the hypocenter is located at an interface between two layers with high velocity contrast.

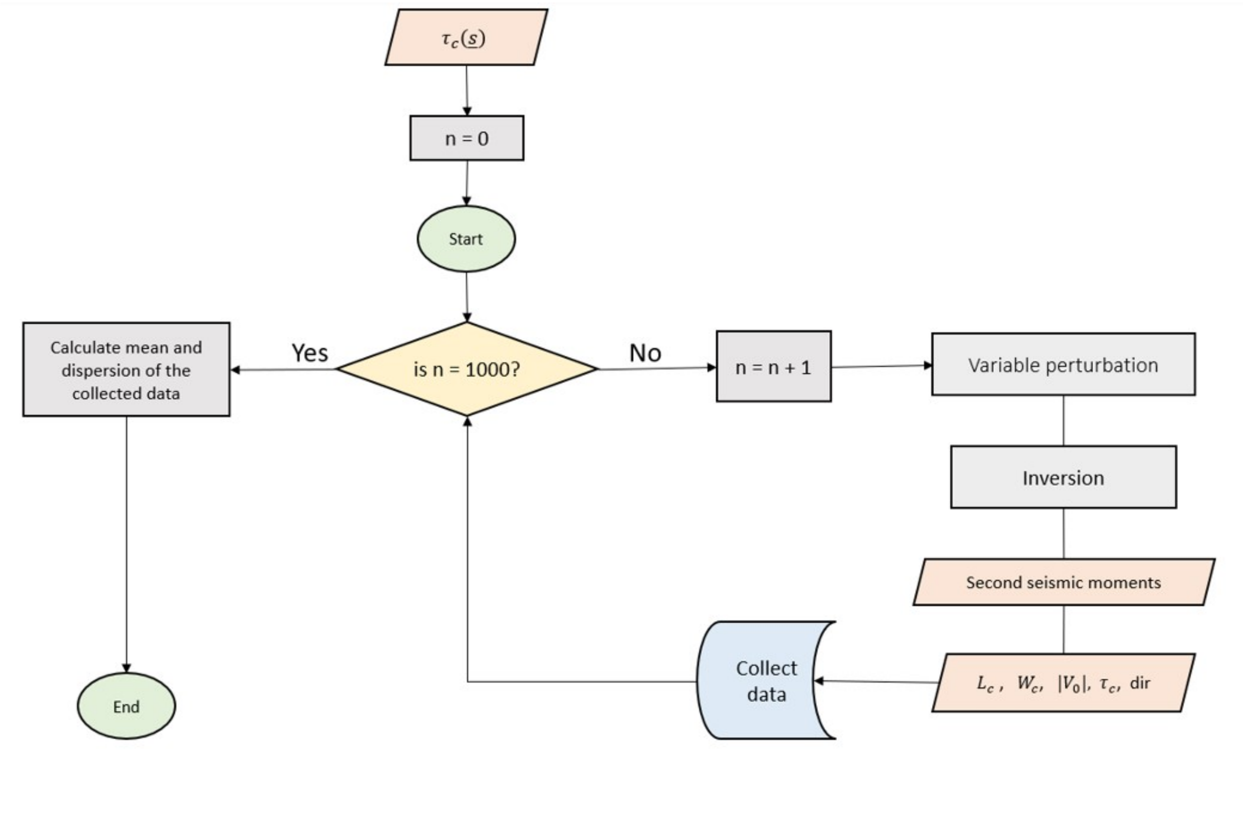

Fig. 2 Flowchart of the perturbation test. For each test, we computed 1000 random station configurations or perturbed input variables (depth, velocity model, focal mechanism, or observed c) with a given standard deviation. Then we performed the inversion and calculated the source parameters and directivity. Finally, we calculated the mean and dispersion of the output variables of the 1000 scenarios.

The values of the directivity depend on the ASTF duration, the choice of velocity model and the focal mechanism (Fig. 3). To ensure good resolution of the fault plane, good coverage of the ray path is critical for both upward and downward waves (McGuire, 2004). The component of rupture directivity along the dip can only be well determined if stations directly above the hypocenter are available, as the seismic rays are nearly horizontal at most other stations.

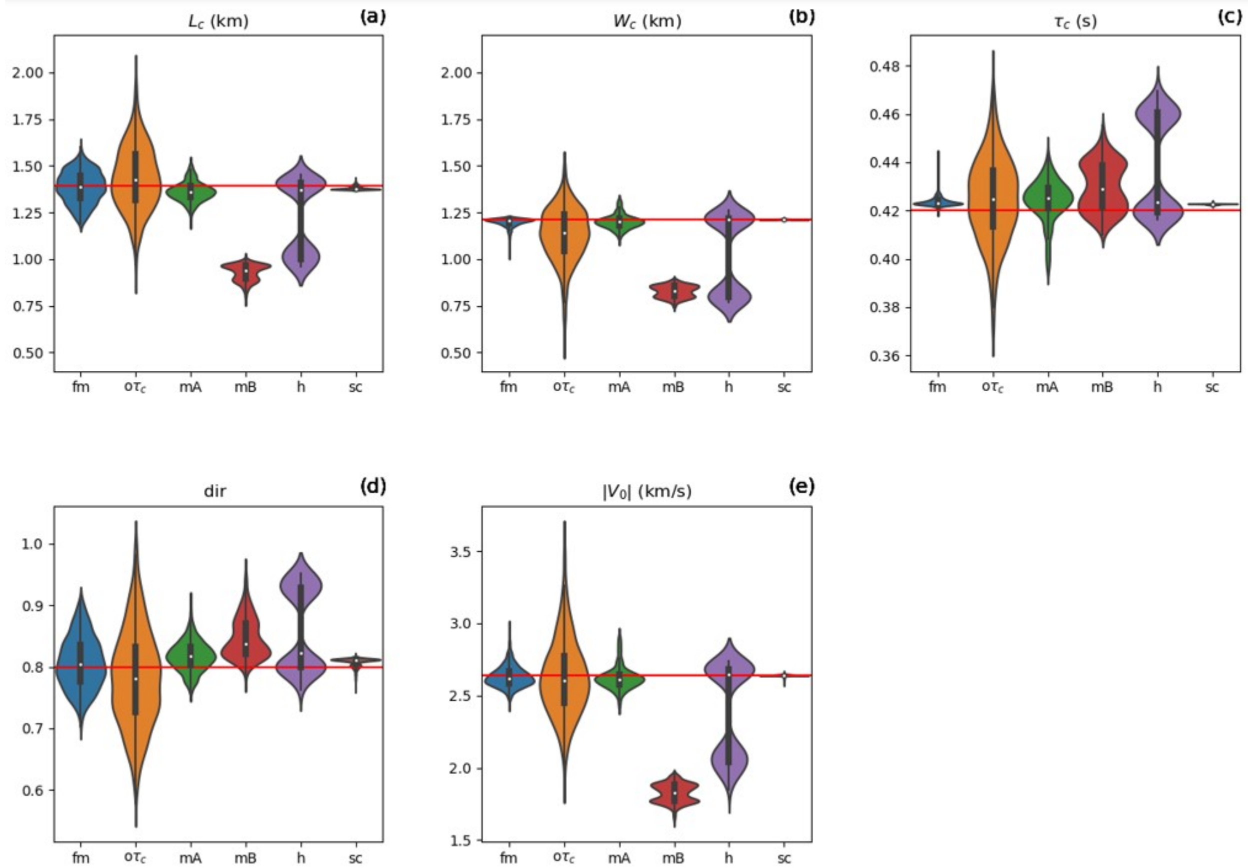

Fig. 3 Violin plots showing the Mean values and dispersions of each output variable resulting from each perturbation test given on the x-axis, i.e., focal mechanism (fm), observed τc (oτc), velocity models (mA and mB, respectively), hypocentral depth (h), and station configuration (sc) for the unilateral scenario. (A–E) represent the solutions for the characteristic length, characteristic width, source duration, directivity and centroid rupture velocity respectively. The y-axis indicates the value of the output variable. The shape of each violin graph reflects the numerical counts of the resulting value. The red line serves as reference, indicating the input value.

	Unilateral rupture					*Bilateral rupture*				
Output variables	*(km)*	*(km)*	*(sec)*	*| (km/s)*	*Dir*	*(km)*	*(km)*	*(sec)*	*| (km/s)*	*Dir*
not perturbed	1.39	1.21	0.42	2.48	0.8	1.38	1.21	0.31	1.13	0.25
Observed (sd = 10%)	1.4	1.13	0.42	2.63	0.78	1.41	1.18	0.31	1.14	0.25
Depth (sd = 1 km)	1.22	1.02	0.44	2.38	0.86	1.20	1.02	0.33	0.81	0.22
Stations' configuration	1.38	1.21	0.4	2.64	0.81	1.39	1.21	0.31	1.12	0.25
Focal Mechanism (sd str = 5°, sd dip = 5°)	1.39	1.20	0.42	2.63	0.81	1.38	1.20	0.31	1.11	0.25
A model (sd = 0.3 km/s)	1.36	1.20	0.42	2.62	0.82	1.37	1.21	0.31	1.10	0.25
B model (sd = 0.3 km/s)	0.93	0.83	0.43	1.83	0.85	0.96	0.84	0.32	0.48	0.15

Tab. 2. Results of the mean of each outcome variable calculated by the perturbation test for the unilateral and bilateral scenarios. For each test case, we report between brackets the standard deviation (sd) applied to the true value.

Application to real case: the Mw 4.6 Central Italy earthquake

The method was then applied to study the Mw 4.6 event of March 2023 in central Italy, using data from the Italian seismic network (RSN (Amato et al., 2008) and the Italian accelerometry network (RAN (Costa et al., 2022)). We compute the ASTFs through the EGF deconvolution using the P and S waves.

We calculated the second seismic moment to obtain information about the directivity and source parameters. The main parameters calculated with this method are the following = 1.16 km, = 0.615, = 0.14 s, = 1.86 m/s, dir = 64, stress drop = 7.37 MPa). The relatively small value of is possibly due to the poor resolution of the vertical component and can be explained by the interaction of two factors: the vertical rupture plane and the small number of stations in the immediate vicinity of the epicenter (< 5 km).

Conclusions

The use of second-moment tensors to determine the source parameters, including directivity, of moderate-magnitude earthquakes could be a valuable tool to improve our understanding of the source dynamics in a given area and to the risk mitigation. One possible application of the second-moments method to small earthquakes would be to identify portions of large faults that produce super-shear ruptures and correlate them with the geology of the fault zone. The second moments method also provides lower constraints on rupture velocity, which can be particularly useful for unilateral ruptures. However, before the results can be interpreted, the resolution limits of the method need to be known due to the possible uncertainties of the parameters used as inputs to the computational procedure.

To overcome the difficulties related to the analysis of noisy signals in the time domain, which can be an important limitation in the calculation of ASTFs and consequently the source duration for low magnitude events, an experimental approach based on the frequency domain is currently being developed. Although the frequency domain deconvolution-based method is currently more time consuming than time domain deconvolution, it can be used in situations where the determination of reliable ASTFs is difficult due to noise, which is often the case for low magnitude earthquakes.

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