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IMPROVED MINIMUM ENTROPY DECONVOLUTION

Abstract. Since its introduction, the Minimum Entropy Deconvolution method has received much attention from Geophysicists because of its ability to compensate for the actual phase of the wavelet. In practice, the method has not proven to be reliable in many cases because of the inherent ambiguity of the solution, and has been replaced by more robust but rough parametric methods.

In this work, a technique for resolving ambiguities is proposed and applied. The technique essentially consists in using parametric methods as a coarse estimate for the proper initialization of the MED algorithm. Furthermore, adaptive and superresolutive features are added to the method. Results on both simulated and recorded traces are presented.

INTRODUCTION

Minimum entropy deconvolution (MED) and the related Zero-Memory Non-Linear (ZNL) deconvolution have been extensively studied in recent years because of their inherent ability to factorize convolutive models of seismic traces making no a-priori assumptions about the phase characteristic of the wavelet (blind deconvolution).

The theory of MED techniques is based on a stochastic model of the reflectivity, i.e. the sequence of acoustic discontinuities in the earth's subsurface. In essence, it is assumed that the reflectivity is a realization of a non-Gaussian random process. In the original works by Wiggins (1978) and Godfrey and Rocca (1981), the reflectivity process is white, while in Walden and Hosken (1985) it is considered as a (1, 1) Arma process. In Jacovitti et al. (1986) a rather general theoretical formulation valid for complex signals is presented.

A very important contribution to understanding the applicability of MED based techniques is given in Rocca and Kostov (1986) where the theoretical performance of the method is discussed.

In practice, many difficulties have been encountered which substantially limit the applicability of MED techniques. In essence, the main problem is the inherent ambiguity of the estimates. In fact, the MED algorithms produce multiple solutions due mainly to the following factors:

- unpredictable noise in the data,
- lack of the convolutional model,
- time windowing.

The multiple nature of MED solutions has been specifically addressed in Nickerson et al. (1987) where it is suggested that the desired solutions should be chosen from a set of possible

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solutions obtained by shifting the window of the deconvolution filter in the MED algorithm.

The MED methods do not provide in se the means for resolving the ambiguities. This problem is common to many estimation procedures where local maxima of a given functional are calculated but coarse (approximate) estimates must be provided to evaluate a proper inital guess.

The first aim of this work is to investigate possible coarse estimates for MED techniques which play the role of fine estimators in a complete deconvolution procedure.

The second aim is to discuss the possibility of including some superresolution features in the MED algorithm. In fact, the band-limited nature of the observed traces substantially limits the resolution of the deconvolution methods, which perform only linear inversion, by definition.

Existing procedures do not in general provide the means for extrapolating the available signal bandwidth. Linear programming and some other multipulse methods model a trace with a series of isolated spikes but in an insufficiently controllable way.

Attempts to perform superresolution by explicitely taking into account statistical properties of the earth layers using Markov models are currently being made (Jacovitti et al., 1989). In the present work, the marginal distribution of the reflectivity (employed in the estimation of the inverse filter) is also employed for spectral expansion. To illustrate these methods, let us first critically review the MED algorithms by discussing their specific features both from a statistical and a deterministic point of view.

THE MED TYPE ALGORITHM

The assumed model for the seismic trace is a convolution of the reflectivity signal r(t) and the wavelet w(t), plus the observation noise $\nu(t)$. The wavelet w(t) represents the unwanted filtering effect of the wave propagation and of the limited bandwidth of the instruments.

$$y(t)=r(t) \; * \; w(t)+\nu(t). \tag{1}$$

More specifically, let us refer to a version sampled every T seconds of the observed signal (1), as actually recorded,

$$y(n)=y(nT), \; n\epsilon I \tag{2}$$

and to the corresponding discrete-time signals (sequences) r(n), w(n), and $\nu(n)$. Our problem is to factorize this signal into an excitation sequence r(n) and a wavelet w(n) by exploiting statistical knowledge about the nature of r(n), and by imposing a defined structure on w(n).

In terms of estimation theory, we can formulate the problem as follows:

Given y(n), find among all possible estimators $\hat{r}(n)$ *and* $\hat{w}(n)$ *the pair minimizing the a posteriori risk.*

The risk is defined on the basis of a suitable error cost function.

Let us refer, for the sake of compacteness, to a vector notation by writing

$$r = [r(0) \; r(1) ... \; r(N-1)]^T, \tag{3}$$

$$y = [y(0) \; y(1) ... \; y(N-1)]^T,$$

and let us assume for simplicity that the noise is negligible.

In Godfrey and Rocca (1981) it has been shown that using a quadratic cost function, and starting from a signal u(n)=y(n)*f(n) (where f(n) is an all zeroes (FIR) initial guess of the inverse filter), the minimization of the risk is accomplished by the following pair of estimation equations for $\hat{r}$ and an all pole (AR) model whose coefficients are the entries of the vector $\hat{f}$:

$$\hat{r}=E_{R/U} \{r/u\}, \tag{4}$$

$$\hat{f}=R_{yy}^{-1}\ R_{y\hat{r}}+s(f).$$

where $E_{R/U}$ indicates the expectation of r conditioned by u, and

$$u = [u(0)\ u(1)...\ u(N+L)]^T, \qquad (5)$$

$$\hat{f} = [\hat{f}(0)\ \hat{f}(1)...\ \hat{f}(L)]^T.$$

R_{yy} is the (L+1)x(L+1) covariance matrix of the input discrete time process (series) and $R_{y\hat{r}}$ is a (L+1)x1 cross-correlation vector between the input series and the estimated excitation.

Based on equations (4), the iterative procedure consists in alternating an estimate of the excitation $\hat{r}$ and of $\hat{f}$ with the evaluation of a new signal u. This is accomplished by updating the filter f with the estimate $\hat{f}$, and by applying it again to the input trace.

The term s(f) takes into account the influence of the old FIR filter f, and is not reported explicitely for the sake of simplicity.

Using superscripts to indicate vectors at the j-th step, and denoting with $F^{(j)}$ the matrix filter built with $\hat{f}^{(j)}$, the MED algorithm is summarized as follows:

REPEAT

$$\left.\begin{aligned} u^{(j)} &= F^{(j)}y \\ \hat{r}^{(j)} &= g_j\ [u^{(j)}] \\ \hat{f}^{(j+1)} &= R_{yy}^{-1}\ R_{y\hat{r}(j)} \\ j &= j+1 \\ c\backslash R_{yy}\hat{f}^{(j)} &= R_{y\hat{r}(j)} \end{aligned}\right\} \qquad (6)$$

UNTIL

where $g_j[.]$ is a function corresponding to the a posteriori conditional estimate in (7). Convergence is attained when the filter shape does not change from one step to the next, as expressed by the last line in (6).

Notice that the term s(f) is ignored in the above iteration. This implies that the MED algorithm corresponds to an optimum strategy only close to convergence where the influence of f becomes negligible (Jacovitti et al., 1986).

The function $g_j[.]$ depends in principle on the whole vector $u^{(j)}$. However, in proximity to the convergence, it becomes a zero-memory function relating only the corresponding entries of the vectors $\hat{r}^{(j)}$ and $u^{(j)}$ (Godfrey and Rocca, 1981).

Before considering this function in detail, let us refer to the observation in Godfrey and Rocca (1981) that the last line of (6) implies a proportionality between the autocorrelation of $u^{(j)}$ and its cross-correlation with the distorted version $g_j\ [u^{(j)}]$. This shows that if convergence occurs then the output signal has the invariance property which characterizes the so-called separable (Bussgang) processes. However, it must be stressed that this property actually refers only to a non-linear function and to the length L of the inverse filter f.

The capability for MED to perform blind deconvolution depends on the statistics $R_{y\hat{r}(j)}$ (l) whose components are the moments

$$R_{y\hat{r}(j)}\ (l)=E\ \{\hat{r}^{(j)}\ (n+l)\ y^*\ (n)\}, \qquad (7)$$

$$=E\ \{g\ [u^{(j)}\ (n+l)\ y^*\ (n)]\}.$$

In the original version of MED (Wiggins, 1978) formulated for real signals, the following is employed:

$$g\,[u^{(j)}\,(l)]=[u^{(j)}\,(l)]^3, \tag{8}$$

which corresponds to a fourth order moment, and was found through the maximization of the varimax norm.

In our Bayesian approach, g[.] is derived from a statistical model of the signal $u^{(j)}$ (n), which can be thought of as the sum of the ideal excitation r(n) and a residual $v^{(j)}$ (n). It has been argued that $v^{(j)}$ (n) behaves like a Gaussian white noise function independent of r(n) near the convergence point (Godfrey and Rocca, 1981). Thus, dropping the indices for simplicity, we may write

$$\hat{r}=g[u]=\int r\, p_{R/U}\,(r/u)\; du, \tag{9}$$

where

$$p_{R/U}\,(r/u)=\frac{p_{U/R}\,(u/r)\;P_R\,(r)}{\int p_{U/R}\,(u/r)\;p_R\,(r)\;dr}\;. \tag{10}$$

The actual shape of g[u] depends on the marginal p.d.f. of the excitation signal $p_R(r)$.

It is important to underline that this function is the central element of the MED process. It allows an extraction of the phase information from the observed process.

Notice that the procedure works only if the process has a non-Gaussian behavior. In fact, in the Gaussian case, g[.] degenerates into a linear function and the MED iterations have no significance.

Let us refer, in particular, to a reflectivity model consisting of a Gaussian mixture

$$p_R\,(r)=\lambda\backslash G\;(0,\;\sigma_1^2)\;+\;(1-\lambda)\backslash G\;(0,\;\sigma_2^2). \tag{11}$$

We obtain the set of non-linear curves displayed in Fig. 1 for complex signals. Very similar curves are obtained for the real case (Godfrey and Rocca, 1981). As expected, these curves extract significant values of r(n) without distortion and suppress the background noise v(n). Furthermore, the p.d.f. of $u^{(j)}$ (n) is given by

$$\begin{aligned} p_U\,(u)&=p_R\,(u)*p_V\,(u)\\ &=\lambda\backslash G\;(0,\;\sigma_1^2+\sigma_{v(j)}^2)\;+\;(1-\lambda)\backslash G\;(0,\;\sigma_2^2+\sigma_{v(j)}^2). \end{aligned}$$

Since the process $u^{(j)}$ (k) is non-Gaussian distributed, it is possible to write the following (non-linear) set of equations:

$$m_u^1=E\{|u|\}=\sqrt{\frac{2}{\pi}}\;[\lambda\;\sqrt{\sigma_1^2+\sigma_v^2}+\;(1-\lambda)\;\sqrt{\sigma_2^2+\sigma_v^2}],$$

$$m_u^2=\sigma_u^2=\lambda\;(\sigma_1^2+\sigma_v^2)\;+\;(1-\lambda)\;(\sigma_2^2+\sigma_v^2),$$

$$m_u^4=E\{u^4\}=3\;[\lambda\;(\sigma_1^2+\sigma_v^2)^2\;+\;(1-\lambda)\;(\sigma_2^2+\sigma_v^2)^2],$$

$$m_u^6=E\{u^6\}=5\;[\lambda\;(\sigma_1^2+\sigma_v^2)^3\;+\;(1-\lambda)\;(\sigma_2^2+\sigma_v^2)^3],$$

where the superscript (j) has been omitted for simplicity.

Taking estimates of m_u^1, σ_u^2, m_u^4 and m_u^6, the above set of non-linear equations could be solved with respect to the unknowns σ_1^2, σ_2^2, σ_v^2, and λ but experience has shown that the poor statistical stability of the estimate of m_u^4 and m_u^6 strongly affects the computed values.

In order to gain effectiveness, the problem is simplified as follows:

- the value of σ_1^2 is taken to be zero. This means that the reflectivity process r(n) is modelled by isolated (sparse) spikes;

- the value of λ is assumed known a priori to be equal to a reasonable value (typically $\lambda=0.9$), and all the uncertainity in the p.d.f. of the process $u^{(j)}$ (n) is given by the signal-to-noise ratio $S = \sigma_2^2/\sigma_v^2$.

Making these assumptions and retaining only the first two (more statistically stable) estimates and the corresponding equations, estimates of σ_2^2 and σ_v^2 are obtained at each j-th stage of the iteration, thus determining the (suboptimal) $g_j[.]$ function.

Using these estimates, the algorithm has been seen to be more stable with respect to the final convergence point, thus drastically reducing (if not completely removing) the oscillatory behavior observed when fixed values of λ and S are taken (Godfrey and Rocca, 1981).

AMBIGUITY OF THE MED ALGORITHM

In the usual version of the MED algorithm, the initial guess of the deconvolution filter is a delta sequence of length L:

$$\hat{f}_0\,(l)=[0\ 0..\ 1..\ 0\ 0\ 0], \tag{12}$$

with the non-zero element in a central position in Nickerson et al. (1987). It was outlined that the choice of this position is a critical parameter leading to different convergence points, and it was suggested that it may be optimized in an empirical way.

In fact, this is a very rough choice giving a coarse estimate of the reflectivity and more flexible initial estimates could be devised. Before discussing this point, let us consider in detail the causes of ambiguity.

Refering to the band-pass nature of the wavelet w(t), we may say that g(u) basically acts as a side-lobe attenuator if one reflector is processed. This means that the objective of the algorithm is to convert the wavelet into one spike as the iterations proceed. The convergence is fast so long as the coarse estimate gives an initial small side-lobe level. If the wavelet has many equal absolute maxima, convergence cannot occur.

When many reflectors are present, the interference between the overlapping wavelets may produce spurious maxima, so that the algorithm is unable to locate the true time positions of the reflectors. In addition, if g(u) has a monotonically increasing derivative then the equilibrium point will correspond to a single reflector located on the absolute maximum of the processed signal. This is a trivial factorization of y(n).

To prevent these problems, the evaluation of the cross-correlation is averaged over several traces, if one wavelet can be assumed as a common convolutive factor. Moreover, the constraint on the deconvolution filter posed by the length L forces the resulting wavelet to have L poles so limiting the possibility of trivial equilibrium points. This emphasises the importance of a careful choice of L in the MED algorithm. Observe that the problem of trivial convergence does not affect, in general, the ZNL algorithm.

Anyway, good behaviour of the MED algorithm depends on the ambiguity characteristic of the residual wavelet after the initial guess. Thus, the best working conditions are attained if in the coarse estimate the side-lobe interferences are minimized. It is evident that both the amplitude and the phase spectrum of the original wavelet must be approximately estimated in order to compensate for large side-lobes. To improve the initial setting of the MED algorithm, it has also been proposed to employ a prediction-error operator as a pre-processor, but the minimum phase character of this operator leads to wrong equilibrium points if the true underlying AR model is non-minimum phase.

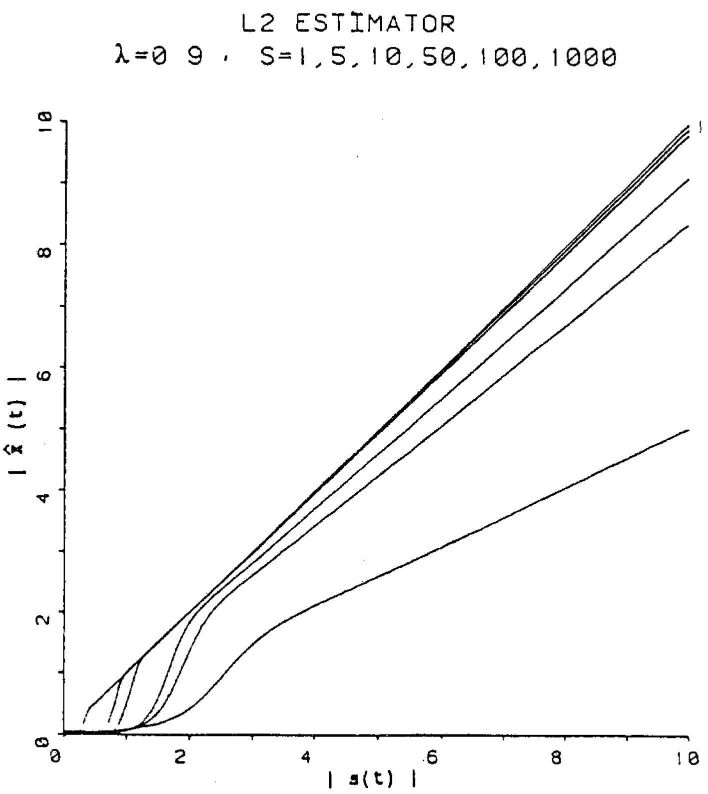

Fig. 1 - Bayesian estimator of the magnitude of the reflectivity based on a quadratic cost function for $\lambda = 0.9$ and $\sigma_1^2 = 0$. The curves are plotted from the bottom in increasing order with respect to the parameter $S = \sigma_R^2/\sigma_N^2$.

THE COARSE ESTIMATE

In principle, we need a rough but stable phase estimate. Experience has shown that polyspectral techniques are not sufficiently stable for applications involving relatively short data sequences. For this reason, we resort to a parametric approach, and in the following, two methods are presented.

The first method is the so-called constant phase correction (CPC) technique. This correction is usually applied after deconvolution in geophysical processing and it is well suited to signals having an amplitude spectrum with a defined peak around the central frequency f_0. In this case, the wavelet can be represented by its complex envelope $\underline{w}\ (t)$ referred to f_0:

$$\underline{w}\ (t)= \frac{1}{2}\ [w\ (t)+j\hat{w}\ (t)]\ e^{-j2\pi f_0 t}, \tag{13}$$

where $\hat{w}$ (t) is the Hilbert transform of w (t).

A constant-phase corrected wavelet is obtained in the form

$$w\ (t)=2\ Re\ [\underline{w}\ (t)\ e^{j(2\pi f_0 t+\phi)}], \tag{14}$$

where ϕ is the constant phase correction term. The aim of the phase correction ϕ is to convert any phase character of the wavelet into a zero-phase (symmetric) behavior. A suitable method

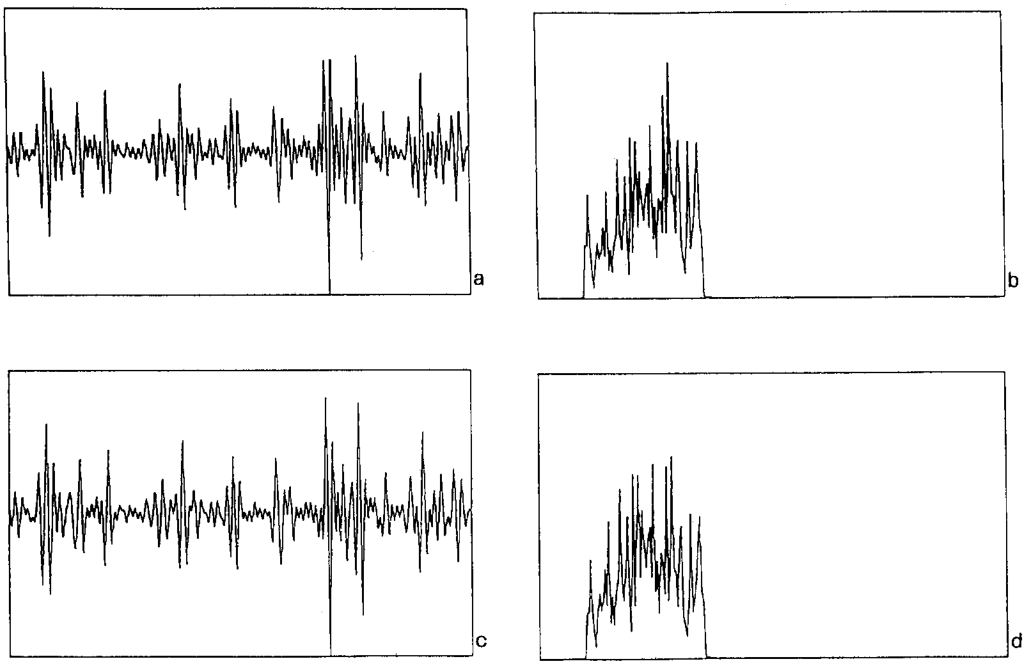

Fig. 2 - The input trace (a) and its spectrum (b) before the estimation procedure. The processed trace after the constant phase correction coarse estimate and the ZNL fine estimate (c), and its amplitude spectrum (d).

for compensating the phase depends on the criterion of maximizing the so-called varimax norm defined as

$$K_w = \frac{\int_T w_\phi^4 (t)\, dt}{[\int_T w_\phi^2 (t)\, dt]^2} \, . \tag{15}$$

In Hosken et al. (1986), it is pointed out that if the excitation series r(n) is independent and identically distributed (i.i.d.) then its kurtosis K_r is related to the kurtosis of y(n)=r(n)*w(n) by the equation

$$(K_y-3)=K_w\,(K_r-3). \tag{16}$$

If r(n) is non-Gaussian, then it is possible to recover K_w from

$$\hat{K}_w = \frac{\hat{K}_y-3}{\hat{K}_r-3} \, . \tag{17}$$

The location of the maximum of $\hat{K}_w$ can be obtained by searching for the maximum of $\hat{K}_y$ versus ϕ. The phase compensated signal is finally given by

$$y(t)=2\, Re\, [\underline{y}\,(t)\, e^{j\,(2\pi f_0 t+\phi_0)}], \tag{18}$$

where ϕ_0 is the phase corresponding to the maximum of $\hat{K}_y$. Notice that the maximization of the kurtosis enhances the non-Gaussian behavior of the signal.

A second method is the so-called Domination Mode (DM). This technique starts from a stan-

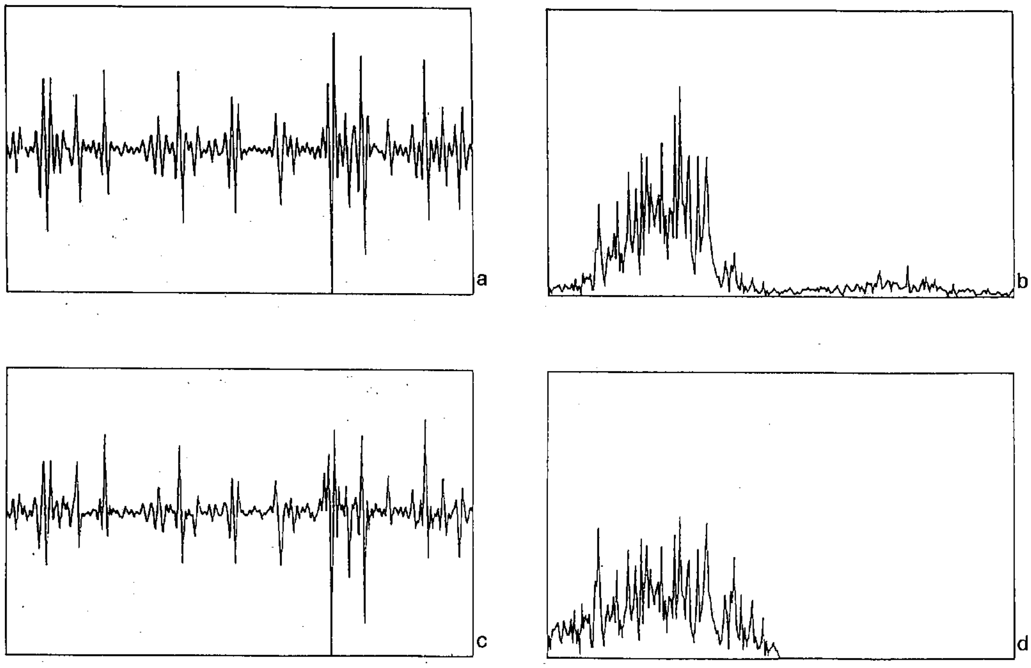

Fig. 3 - Results of application of the non-linear function to the coarse estimate of the same trace as in Fig. 2 (a), and the relative "full-band" amplitude spectrum (b). The processed trace after the band-limited ZNL fine estimate (c), and its amplitude spectrum (d).

dard minimum phase AR estimate, $\frac{1}{A_M(z)}$. Applying $A_M(z)$ to the signal y(n) gives an amplitude compensated signal $\eta(n)$. A rooting is then performed to individuate the pole p_M corresponding to the dominant mode. An all-pass filter is defined by

$$H(z) = \frac{1 - p_M z^{-1}}{p_M^* - z^{-1}} ,$$

and applying this filter to the signal η (n) gives a partially phase compensated signal η' (n). The kurtosis K_η and $K_{\eta'}$ are then computed to decide whether η (n) or η' (n) is less Gaussian. The selected signal is finally put into the MED algorithm. In the case of real signals, the same processing applies to a a pair of conjugate poles. Moreover, if a set of dominant modes is identified, then the procedure can include all the possible signals obtained by passing η (n) through the associated all-pass filters. For M poles (or pairs of conjugate poles), 2^M signals must be compared. Both the coarse estimates here are relatively expensive from the computational point of view. On the other hand, they give a substantial reduction in the number of iterations in the subsequent MED algorithm. A typical reduction factor is $\frac{1}{3}$.

The two methods can be cascaded to obtain further phase correction before MED. However, it has been found that the DM method is often sufficient to resolve some cases where the original MED approach does not work.

SUPERRESOLUTION

The second major obstacle which prevents successful practical applications of the non-

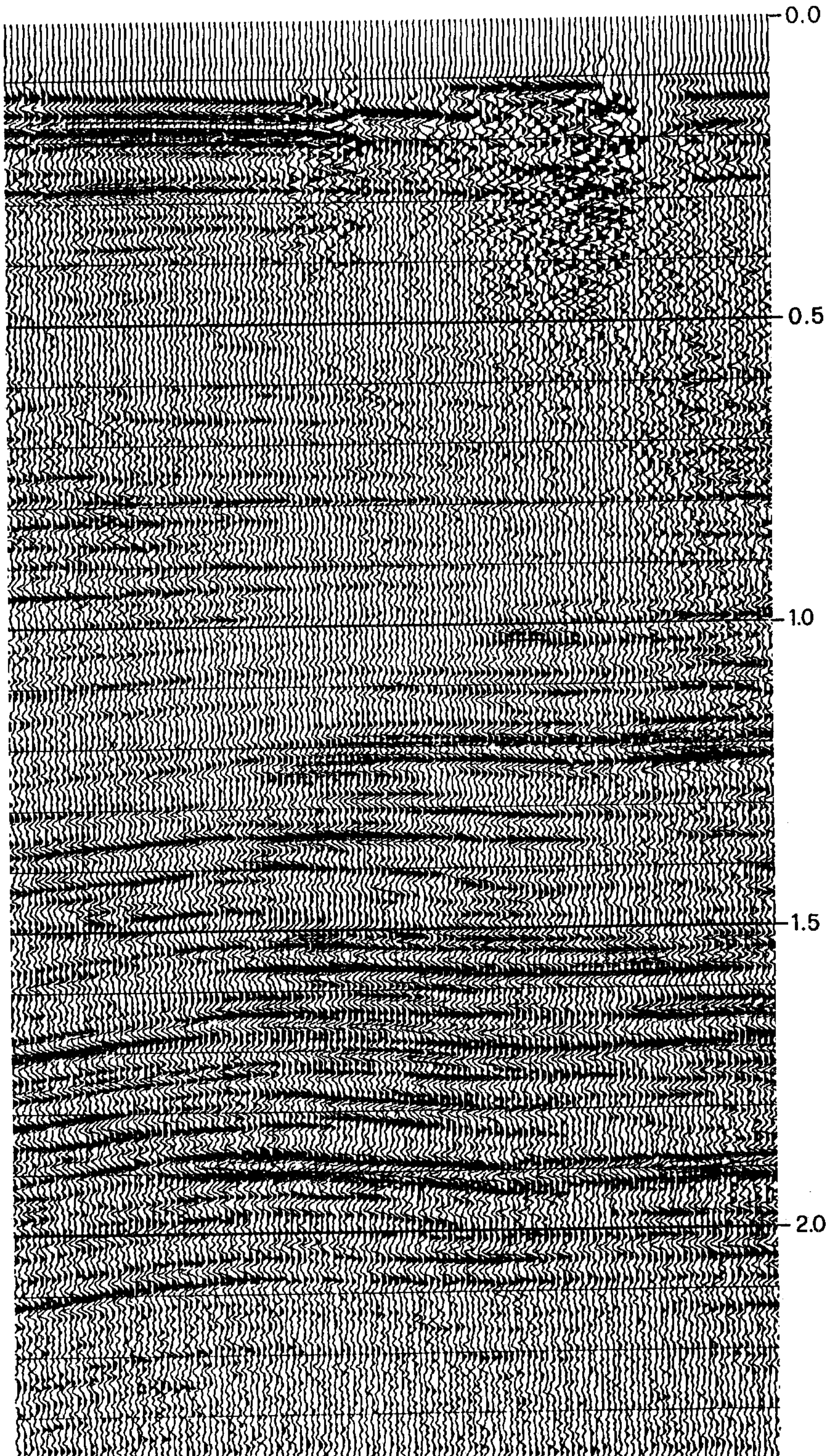

Fig. 4 - Original stacked section of marine seismic data.

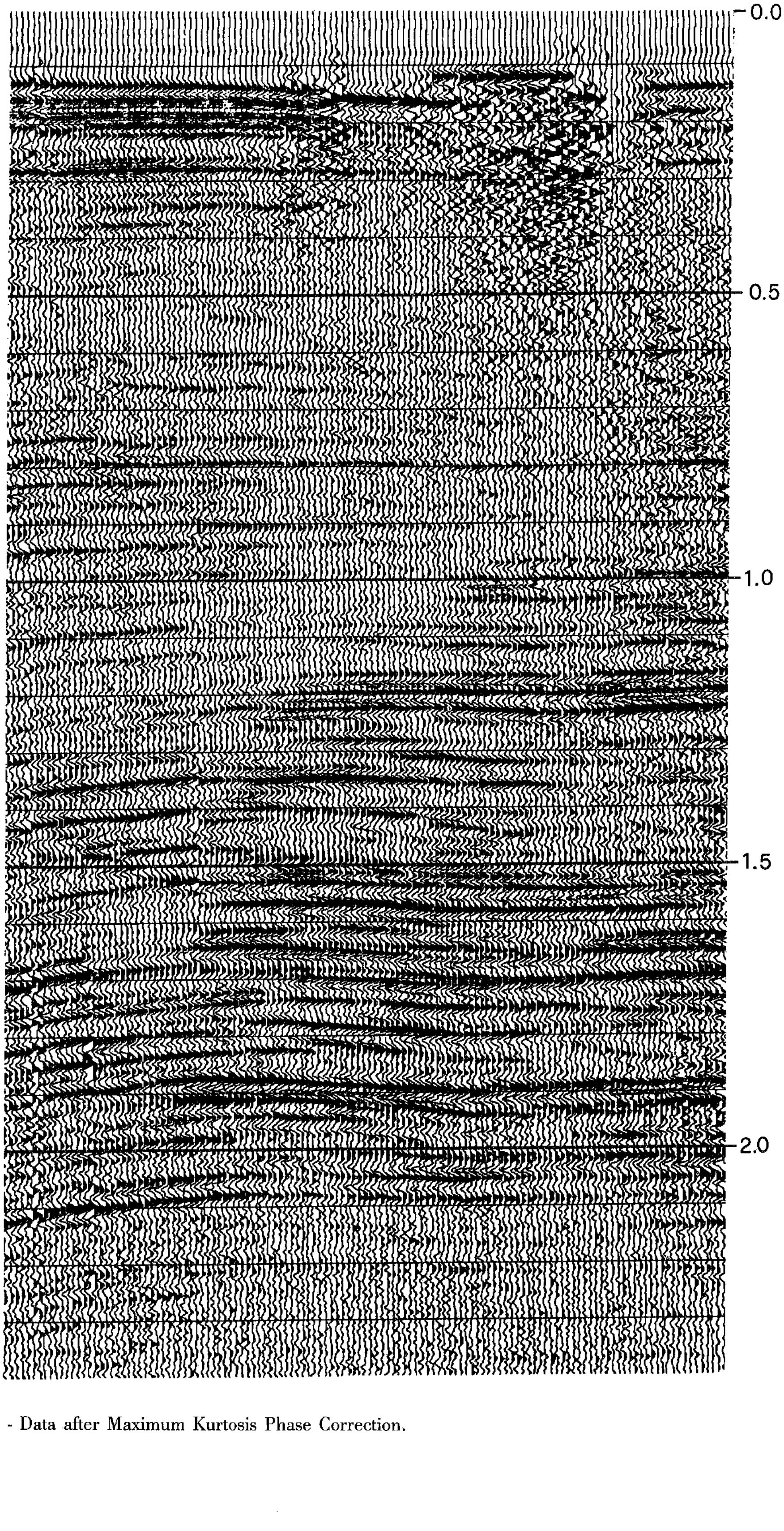

Fig. 5 - Data after Maximum Kurtosis Phase Correction.

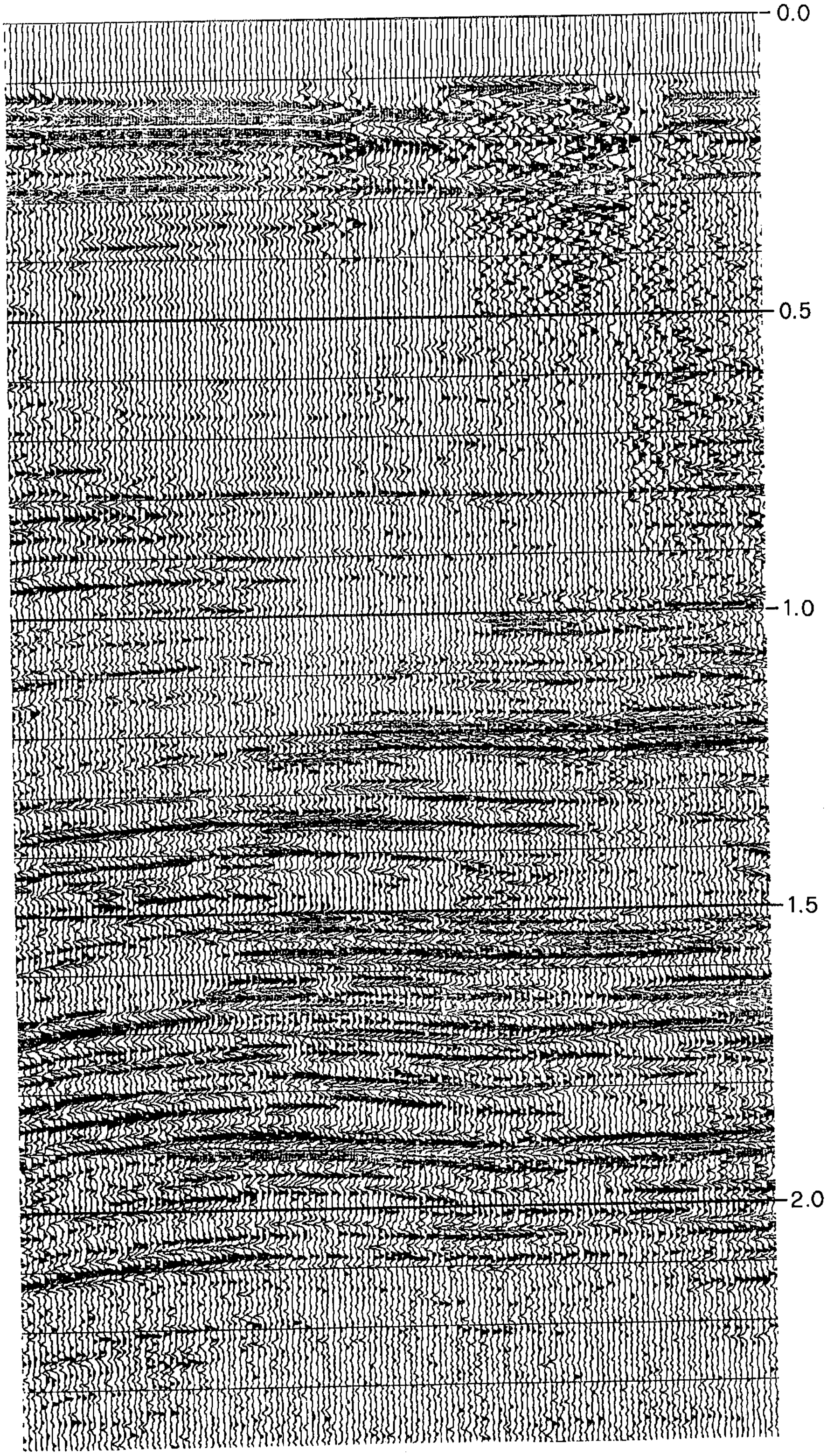

Fig. 6 - Minimum Entropy Deconvolution applied to MKPC section.

minimum phase blind deconvolution is the finite bandwidth of the measured signals. In other words, the measurements produces a fall-off in the spectrum which is much sharper than that allowed by the AR model assumed in the MED or ZNL algorithms.

Much effort has been given recently to extrapolating the spectrum of the signal beyond its bandwidth in order to obtain a full-band deconvolved output. We recall that superresolution is possible if some a priori information about the data is available. For instance, linear programming techniques assume that a certain percentage of the reflectivity samples is zero.

Referring to the MED algorithm, it is known that the finite bandwidth of the input signal causes an ill-conditioned behavior of the normal equations, which can be stabilized by adding a constant value to the main diagonal of the covariance-matrix. This in turn implies a resolution reduction, i.e., the opposite effect to our goal.

Alternatively, one could assume as estimate the output of the instantaneous non-linear function instead of the deconvolution output. In the ZNL method, this causes a specrum expansion related to the suppression of small reflectors. However, an appreciable increase in resolution is not produced in this way.

It has been found that a more effective method is to apply the non-linear function to the coarse estimate and to employ the resulting signal as input to the MED (ZNL). The resulting signal is full-band and is processed by the MED (ZNL) algorithm without inversion problems. The only artifacts expected are formed by the suppression of small events, in a similar way to the linear programming approach. However, this effect can be controlled easily and a satisfactory trade-off between resolution and suppression can be obtained but regulating the initial non-linear function.

The basic behavior of this algorithm is shown in Fig. 2. In Fig. 2a, a band-pass filtered version of the input signal is displayed beside its ideal spectrum (2b). Applying the coarse estimate (constant phase correction) cascaded with the ZNL algorithm gives the output in Fig. 2c and the corresponding spectrum (2d). Let us now apply the non-linear function to the coarse estimate before entering it in the fine estimator (3a). The new input spectrum (3b) exhibits spreading of the band-energy over the whole frequency range. In order to avoid artifacts due to the reconstruction of very high frequencies, the final deconvolution process was done over the first half band only, and the deconvolved signal is depicted in Fig. 3 c beside the associated spectrum (3d). The superresolution effect is clearly visible.

EXPERIMENTS ON MEASURED TRACES

The MED (ZNL) programs developed and the coarse estimation procedures were applied to some seismic traces in order to assess their validity under different environments.

Provided that the lack of robustness in MED has been substantially overcome, it is possible to apply non-minimum phase wavelet estimation in a variety of situations. Depending on the problem at hand, we can apply more or fewer parameters to describe the wavelet. Thus, we establish a trade-off between the bias and the variance of the estimates. The most simple and robust processing is constant phase correction, which is based on the estimation of one parameter.

In Fig. 4, a seismic section obtained from marine recording from the Adriatic sea using .002 sec. as sampling rate and 48 meters as receiver interval is displayed.

A constant phase correction based on the Kurtosis maximization gives the result shown in Fig. 5. This was obtained by estimating the total phase correction on all subsections of the single trace under consideration and then subtracting it without changing the amplitude. The stationarity hypothesis was strenghtened both in time, by taking 512 samples at a time, overlapped by 25 per cent to smooth side effects, and in offset, by averaging the 24 traces around that processed. The reflection events are visually more evident, even if no resolution gain is achieved.

Application of ZNL to the phase corrected section, with a 17 sample inversion operator, gives the output displayed in Fig. 6. The windowing approach for this stage is identical to the former. The spectrum is still averaged, but because MED is sensitive to changes in amplitude, we chose to consider only a small number of adjacent records - two for the data considered here. The reflection events are now much sharper while very small events tend to disappear, as expected. However, no loss of information due to the superrésolution seems to occur.

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