Migration in the angle domain: theory and practical aspects

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ABSTRACT Kirchhoff prestack depth migration in the angle domain helps in the reduction of acquisition footprints, in true amplitude migration, in Amplitute Versus Offset or Angle analysis and in migration velocity analysis. Scattering angles replace surface related acquisition coordinates in the Kirchhoff summation. The regularization of illumination is performed with respect to scattering angles and illumination dip angles. At each depth point, the coverage in the angular domain is equalized by an explicit regularization through binning and hit counting. After a review of migration in the angle domain, we examine some practical aspects of implementation, namely those that mostly impact performances and effectiveness: memory, poor illumination and sampling of integration domain. The most critical factor is memory. We suggest a strategy, based on the computation of an off-line, coarse sampled, hit-count panel, to face the increase in memory requirement. We successfully limit noise amplification by using a damping factor.

1. Introduction

Migration in the angle domain has been proposed with success as a tool for the migration velocity analysis (Brandsberg-Dahl *et al*., 1999). Xu *et al*. (1998) showed that CRP gathers in the scattering angle domain, contrarily to the offset domain, do not suffer from multi-pathing in complex media. Moreover, in order to restore the reflector amplitudes, that is true amplitude migration, and for purposes of Amplitute Versus Angle (AVA) analysis, it is essential to consider both of the acquisition footprints and the effects of the wave propagation through the medium. In the Kirchhoff migration, this is done through a proper weighting of the traces. The Beylkin determinant (Beylkin, 1985) is part of this weight. The Beylkin determinant is constant in the angle domain (Albertin *et al*., 1999). This allows us to treat the problem of computing the Beylkin determinant through an explicit equalization at the depth point (Audebert *et al*., 2000, 2002, 2003), in particular by performing binning and hit-counting in the angle domain.

Theoretical aspects of migration in the angle domain are well investigated; we analyze some practical aspects related to its actual implementation namely those that impact mostly on performances and effectiveness: memory, poor illumination and sampling of integration domain. The first problem arises from the difficulties related to the enlarged requirement of memory that makes the brute force approach unaffordable. We suggest some expedients for solving this problem. Another aspect we investigated is related to the negative effect of equalization in poorly illuminated zones. The approach of performing binning and hit-counting involves some issues related to the tessellation of the integration domain.

First, we describe which angles are involved. After a brief review of migration and regularization in the angle domain, we examine the implementation issues.

2. Migration in the angle domain

Kirchhoff migration algorithms represent, today, the state of the art for industrial PreSDM software, because of their precision, efficiency and flexibility. Despite this success, in case of complex velocity models, and irregular 3D acquisition geometry, the limitations of Kirchhoff migration become visible. Some of these limitations are intrinsic to the methodology, e.g. those caused by the high-frequency approximation. Several problems arise from the actual implementation instead. Migration should not be performed simply by a weighted diffraction stack, as if the integral was regularly sampled. Acquisition geometry and propagation in the subsurface cause irregular sampling. Introduction of the angle domain at the depth point allows us to this problem as a simple equalization of illumination.

The principles of migration in the angle domain are well explained in Audebert *et al*. (2000, 2002, 2003). Source and receiver positions identify the slowness vectors **S** (incident slowness) and **R** (scattered slowness) at the depth point (Fig. 1). The **S** vector has the direction of the ray from the source position to **x** while the **R** vector has the direction of the ray from the receiver position to **x**. The magnitude of these vectors is 1/c(**x**), where c(**x**) is the background velocity.

Vector **D** is the sum of these two vectors, and it represents the gradient of the total traveltime from the source to the depth point and to the receiver. The direction of **D** is defined by the unit vector $\mathbf{u_D}$. This vector $\mathbf{u_D}$ has the following properties:
- it is normal to the isochronal surface;
- it is the DIP of the migration operator;
- it is the vector that describes the hypothetical specular reflector at the depth point.

The vector $\mathbf{u_D}$ describes the unit hemisphere. The migration formula appears as an integral over this hemisphere. Once the vectors **S**, **R** and **D** are defined, it is possible to describe them through four angles that can have an easy interpretation.

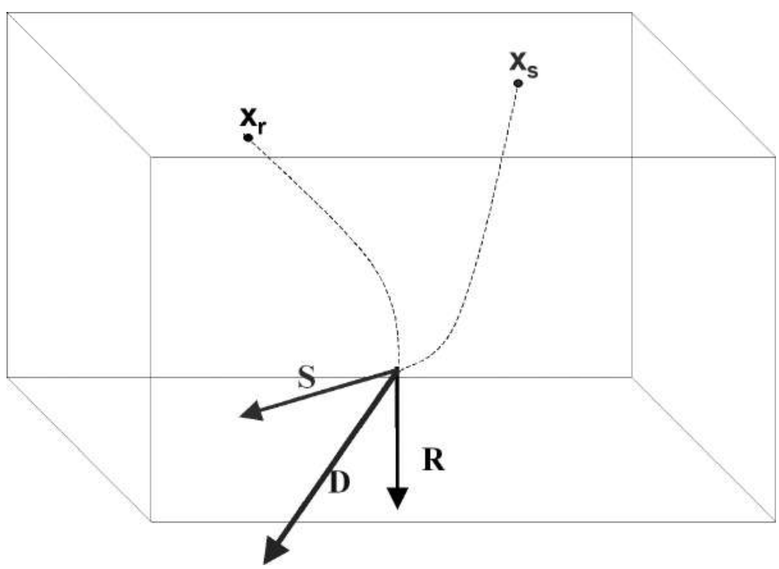

Fig. 1 - Slowness vectors at the depth point.

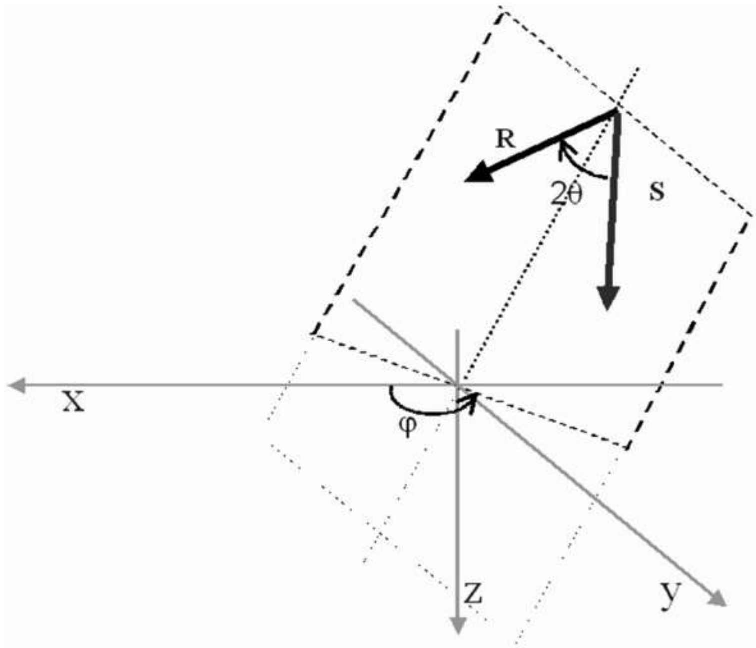

Fig. 2 - Scattering angles.

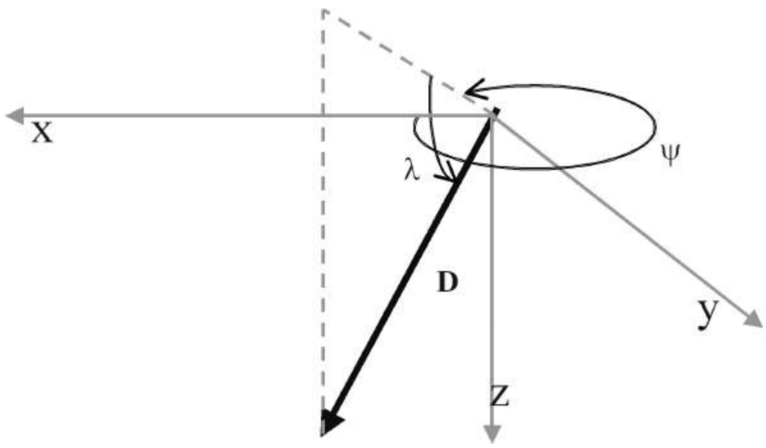

Fig. 3 - Illumination angles.

Two angles are related to the incidence angle and are named scattering angles (Fig. 2):
- θ: half the aperture between the slowness vectors;
- φ: azimuth of the plane that contains the slowness vectors.

Two angles describe the illumination dip vector (Fig. 3):
- λ: elevation of the illumination dip vector;
- ψ: azimuth of the illumination dip vector.

The scattering angles θ and φ have a simple interpretation. The angle θ plays the role that is assumed by the offset on surface coordinates. This angle is related to the incidence angle: it is the incidence angle if there is a reflecting surface in the direction of $\mathbf{u_D}$. The angle φ plays the role of the azimuth on surface coordinates. Once the scattering angles are fixed, the migration consists in the integral over dip angles, for each depth point, i. e. over the unit hemisphere:

$$I(\mathbf{x},\vartheta,\varphi) \propto \iiint j\omega W(\mathbf{x},\vartheta,\varphi,\lambda,\psi)U(\mathbf{x},\vartheta,\varphi,\lambda,\psi,\omega)e^{-j\omega\tau}d\omega d\lambda d\psi \quad (1)$$

Here $I(\mathbf{x},\vartheta,\varphi,)$ is the migrated image $W(\mathbf{x},\vartheta,\varphi,\lambda,\psi)$ is a weight function $U(\mathbf{x},\vartheta,\varphi,\lambda,\psi,\omega)$, is the seismic data set, τ is the sum of the traveltimes from source and receiver to the depth point **x**. The weight function contains the Beylkin determinant that is the Jacobian of the transformation from surface coordinates to the depth point. This Jacobian is constant in this domain: this is the most natural domain for this integration.

3. Regularization of illumination

The migration integral is approximated by the discrete summation:

$$I_{weq}(\mathbf{x},\vartheta,\varphi) = \sum_{\lambda,\psi} M(\mathbf{x},\vartheta,\varphi,\lambda,\psi) \quad (2)$$

$M(\mathbf{x},\vartheta,\varphi,\lambda,\psi)$ is the total migration contribution in the angular bin defined by $\mathbf{x},\vartheta,\varphi,\lambda,\psi$ and $I_{weq}(\mathbf{x},\vartheta,\varphi)$ is the migrated image.

If the integration domain, that is the hemisphere described by the illumination vectors, is tessellated with bins of equal area, the integration is correctly weighted only if there is just one sample for each $\Delta\lambda\Delta\psi$ area element (bin). We use the arithmetical average of the samples that are in each area element as the correct contribution to the summation, this procedure is the regularization of illumination. From a practical point of view, we regularize by counting the contributions in each bin and dividing the summation by the number of contributions:

$$I_{eq}(\mathbf{x},\vartheta,\varphi) = \sum_{\lambda,\psi} \frac{M(\mathbf{x},\vartheta,\varphi,\lambda,\psi)}{C(\mathbf{x},\vartheta,\varphi,\lambda,\psi)} \quad (3)$$

$C(\mathbf{x},\vartheta,\varphi,\lambda,\psi)$ is the hit-count in the angular bin defined by $\mathbf{x},\vartheta,\varphi,\lambda,\psi$ and $I_{eq}(\mathbf{x},\vartheta,\varphi)$ is the equalized image. Now the $\Delta\lambda\Delta\psi$ factor can be neglected as a scale factor. This procedure is equivalent to the application of the Beylkin determinant (Beylkin, 1985; Albertin *et al.*, 1999) but is more robust and can be used for any kind of acquisition geometry. Fig. 4 shows the image of a horizontal reflector from a regular acquisition with a hole (obtained with a coarser sampling of sources and receivers) before (top) and after (bottom) regularization. Fig. 5 shows the illumination (hit-count panel) for the geometry of the data set migrated in Fig. 4 and for $\lambda<5°$ (illumination of flat reflectors): the hole in the illumination is evident in the central zone.

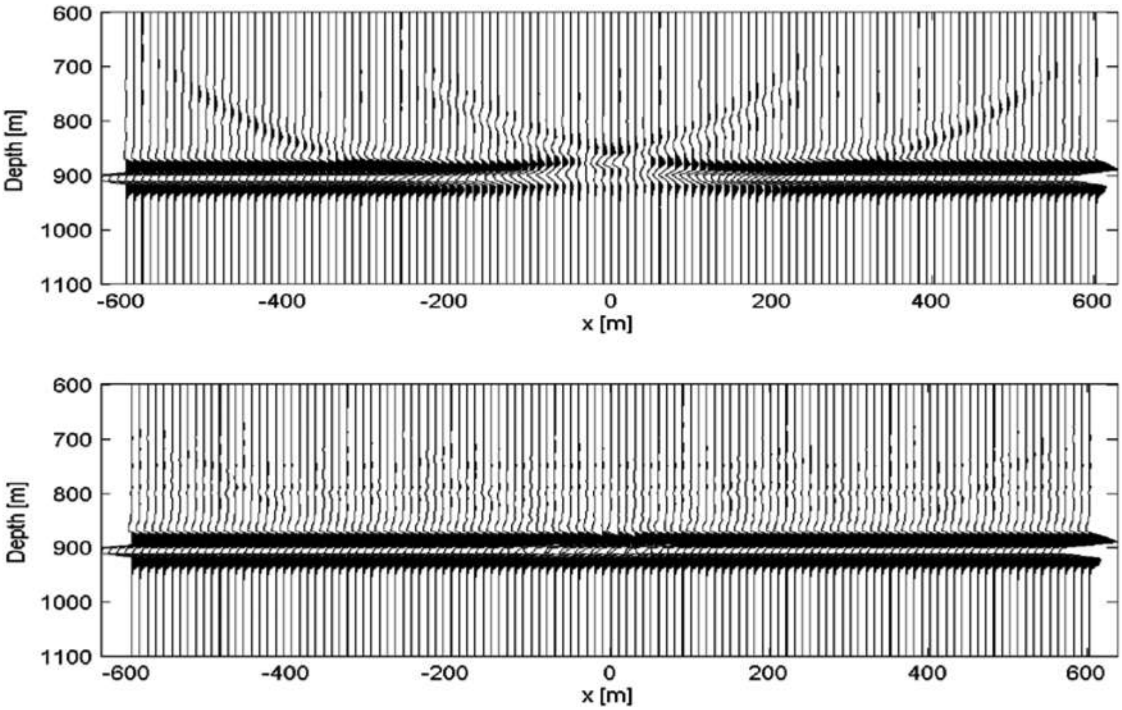

Fig. 4 - Migration of a horizontal reflector, synthetic data set, and regular acquisition with missing data. Top: without regularization. Bottom: with regularization.

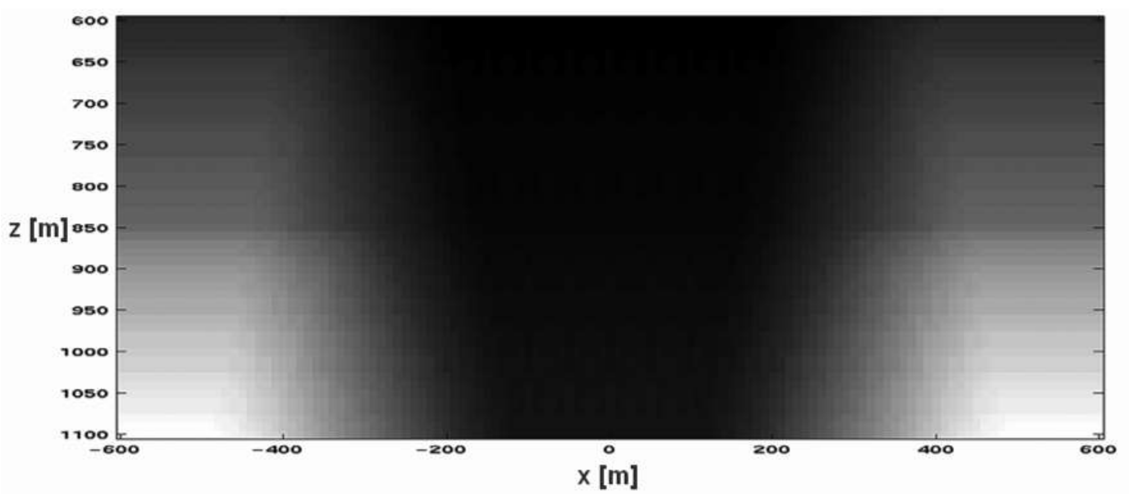

Fig. 5 - Hit count panel (stack for λ <5º).

4. Issues related to memory requirement

Before regularization and summation over dip angles, the output of the migration process is a 7-dimensional matrix (beyond the standard three geometrical dimensions the approach needs other four additional dimensions for the angular coordinates). This implies a huge increase in memory with respect to the standard migration. Let the target be a cube of a 1 km edge with a 25 m sampling step: the memory requirement to store the migration output is 256 kB. Migration in the angle domain could require dozens of GB**.** The brute force approach is unaffordable.

Some solutions allow the solving of these problems:

- **neglecting angle φ:** by assuming an isotropic medium, the AVA response is not influenced by the angle, so it is possible to avoid the classification with respect to this parameter;
- **off-line hit-count:** the hit-count can be computed before the migration, it halves the memory requirement and it allows us to know which angular bins have to be taken in account before the actual migration;
- **coarse sampling**: the hit-count can be computed on a coarser grid and then interpolated;
- **equalization during migration**: the equalization can be implicitly done during the summation.

With these expedients, the additional memory is that required to store the coarse sampled hit-count hypercube. In the example mentioned above, the order of magnitude of the memory required becomes of a few MB. This makes the method affordable.

5. Effect of noise

The regularization is obtained by dividing each angular contribution for its hit-count value, thus it suffers from division by small numbers (noise amplification). This risk is more critical in the zones of poor illumination, potentially the ones that mostly benefit from regularization. The solution is the application of a damping factor. The choice of the damping factor value has to be a trade-off between the amplification of noise and the regularization of illumination. Fig. 6 shows the image obtained from a synthetic data set with white noise superimposed. We can clearly see how the regularization allows the recovery of continuity of the reflector, at the price of a strong increase of noise in the badly covered zones (middle). An appropriate choice of damping recovers the continuity of the reflector avoiding a large increase of noise (bottom).

6. Sphere tessellation

Regular tessellation of arbitrarily fine resolution is not possible on the sphere: an irregular tessellation must be used to have an equal area. The simplest way of implementing an equal area tessellation is to use a regular, angular step in the azimuth dimension and a variable step in the elevation dimension. The shortcoming of this strategy is that in many practical cases the migration contributions are gathered in the small elevations and do not allow a correct discrimination of small dips. A possible strategy to solve this problem consists in placing the poles on the horizontal plane; this allows a more regular sampling near the top of the sphere (small elevations). It is also possible to abandon the constant area constraint, by using a constant

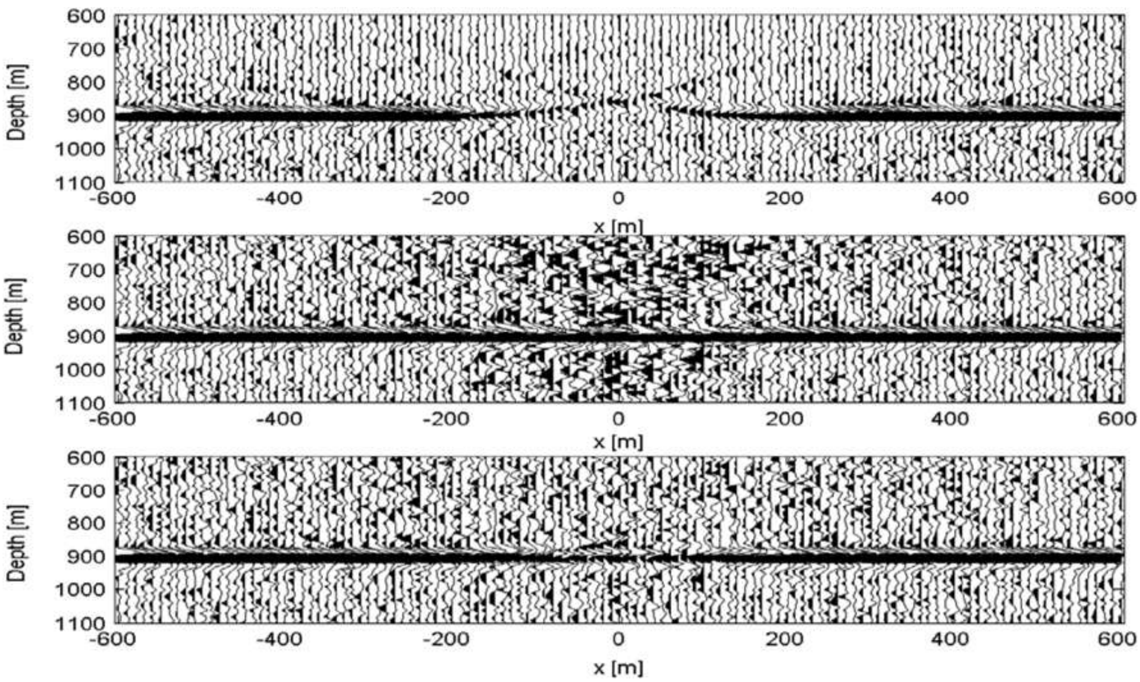

Fig. 6 - Results of the migration with hit-count interpolation.

angular step, and to correct the result by a term proportional to the bin areas. However, this choice is more critical because the corrective factor tends to infinity, for small values of elevation steps, near the pole.

7. Conclusions

We studied some issues related to the implementation of migration with regularization in the angle domain. This method is effective for velocity analysis, Amplitude Versus Offset/AVA analysis and true amplitude migration but it involves some implementation difficulties. To solve some of the issues related to the increased memory requirement we calculate the hit-count before migration on a coarser grid. We contain the effect of noise amplification in zones of poor illumination using a damping factor. Several implementation schemes of the sampling of the integration domain have been studied.

The use of geographical-like coordinates, with or without the constant area constraint, has a deficit in handling the small dips. The solution that makes use of a constant area geographical-like coordinates, but with the poles placed in the horizontal plane better handles these issues while preserving a simple implementation.

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