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SPACE (3-D) AND SPACE-TIME (4-D) ANALYSIS OF AFTERSHOCK SEQUENCES: THE FRIULI (NE ITALY) CASE

Abstract. Space-time hypocentral coordinates from a fixed number of aftershocks in a time window sliding along the sequence may be used to compute a set of geometric moment matrices thought to represent 4-D rupture hyperellipsoids. Time variations of the hyperellipsoids size and shape, bound to the time variations of the shocks space-time distribution clearly mark the sequence beginning and the onset of different subsequences. The four principal axes of the hyperellipsoids, projected onto 3-D space, depict the seismogenic fault orientation, slip and rupture propagation directions, and the layout of the possible stack of the same-order faults. Discrimination of the single subsequences which constitute a seismic series and the successive directional analysis allow the construction of a reliable evolutionary fracture model.

INTRODUCTION

The relationship between earthquakes and faults was first established by Reid (1911). Since then, seismological data are commonly used to infer seismogenic first-order rupture orientations and their mechanism, i.e. slip direction. In fact, the fault plane solution technique (Byerly, 1926) is widely adopted, though its applications are limited to large events recorded by a sufficient number of stations, or to sets of smaller ones which exhibit the same focal mechanism. Even then, the solutions, commonly obtained using the polarities of first arrivals of P-waves only, cannot discriminate between the two nodal planes so revealing the orientation of the active fault.

An additional simple technique for finding the orientation of seismogenic faults, if not that of the slip associated with them, is to find the best-fit straight lines through the hypocentral projections onto variously oriented cross-sections or, even better, to fit a plane through the 3-D hypocentral cloud (e.g. Fehler et al., 1987).

However, the latter method does not appear to be an adequate representation of an earthquake source geometry which has finite dimensions and typically results from multiple fractures which, in terms of their seismogenic role, may be thought of as being infinite in number (Kagan and Knopoff, 1980, 1981; Kagan, 1981a, 1981b, 1982). Furthermore, several studies based on the principles of self-similarity and on the concepts which attach a spatial, geometrical meaning to fractal dimensions (Sadovskii et al., 1984; Hirata et al., 1987; Smalley et al., 1987; Yamashita and Knopoff, 1987, 1989) suggest that, in many cases, seismogenic rupture mechanisms tend to involve, rather than single surfaces, entire rock volumes, tentatively identified as those marked by the cloud of hypocentral locations (Båth and Duda, 1964; Ranalli, 1969; Ranalli and Scheidegger, 1969; Scheidegger, 1982).

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Vectors connecting subsequent pairs of aftershock hypocentres are not randomly oriented, and neither are the poles normal to the planes determined by three hypocentres; on the other hand, their preferential directions are not always parallel or perpendicular to the best-fit line through the whole or part of the cloud. This suggests that analyses of the time evolution of hypocentral locations might disclose features connected with possible, higher-order discontinuities.

The Principal Parameters Method (PPM) was devised (Ebblin and Michelini, 1986) to extract the preferential propagation directions from hypocentral data, and its application proved to be successful in revealing the orientation of planar features which appear to be consistently parallel to one of the nodal planes of the fault plane solutions (Ebblin and Michelini, 1986; Michelini and Bolt, 1986; Tselentis et al., 1989). Partial application of the PPM by Fehler and Johnson (1989) to groups of shocks statistically related to different planes also gave comparable results.

The method assumes that a proper description for the tendency of a sequential set of points to follow in particular preferential directions may be given by their geometrical moments of inertia. The PPM merely considers the first- and second-order moments of sets of hypocentres to which a unit mass is attached, thus providing only the first- and second-order approximations of their distribution, the former defining the centre of gravity of the set and the latter the symmetrical part of the dispersion around it. The second-order moment matrices have also been called scatter matrices (Ebblin and Michelini, 1986), spread matrices (Michelini and Bolt, 1986), or variance-covariance matrices (Tselentis et al., 1989). The ellipsoids representing the quadrics, the coefficients of which are the elements of the matrices, were named rupture ellipsoids.

Hence, the PPM postulates that, in a focal region, seismogenic slip is not confined to a single continuous surface but, rather, that it involves a whole number of discontinuities of various orders. The 3-D ellipsoids are presumably representative of the active volume of the rocks, and their principal axes might be related to the spatial fault parameters. The PPM involves sliding a time window of fixed width along the observed sequence of events, arranged in temporal order, thus obtaining a picture of the time evolution of the rupture process. It is interesting to note that a sequence characterized by two very different concentrations of ellipsoid orientations was reduced to displaying only one when the sequence was randomized with respect to time.

This fact proves the close link between the time and space distribution of foci, and suggests a different way of considering earthquakes. In fact, they may be represented by point events in a five-dimensional space-time-energy continuum. A complete analysis of earthquakes must, therefore, consider the distributions and correlations of all the five parameters characterizing a seismic event: latitude, longitude, depth, origin time, and energy (Udias and Rice, 1975).

The purpose of the present work is to attempt such a multi-dimensional approach to the seismic process, in order to deal more effectively with its complexity. With this aim, the PPM is extended to four dimensions by introducing time into the analysis as an independent variable, and thus to evaluate the shock distribution in the space-time domain. The rupture hyperellipsoid's shape and size variations reflect any changes in the shock clustering in space as well as in time. Furthermore, the PPM in 4-D yields four principal axes, the projections of which onto 3-D space are no longer necessarily orthogonal, thus providing a more versatile model for seismogenic events.

THE METHOD IN FOUR DIMENSIONS

The PPM is based upon the relative distances of a group of foci in their various directions from the centre of gravity of the set defined by their coordinate means.

Obviously, while performing the 4-D space-time analysis, the time dimension has to be made homogeneous with the spatial ones. The most obvious ways to accomplish this is either by applying a velocity factor to the time coordinates or by normalizing the coordinates to make them dimensionless. Both were tried. The results of the latter procedure are shown here because the former required too great velocity variations to obtain all four hyperellipsoid principal axes with comparable lengths.

In addition, the normalized time interval reciprocals are chosen to represent the fourth dimension, in agreement with the aftershock concept (Gardner and Knopoff, 1974), according to which, may be regarded as belonging to the aftershock sequence also events not close to the main shock, if immediately after it in time, whereas only the closest shocks after a longer period. The alternative use of the normalized time intervals, instead of their reciprocals, showed less striking but similar results.

The set of parameters $L=[l_{ij}]$ are defined such that

$$l_{ij}=l_{ji}=m^{-1}\sum_{k=1}^{k=m}(x_{ki}-\bar{x}_i)\,(x_{kj}-\bar{x}_j),$$

where m is the number of foci, with $i, j=1, \ldots, 4$ in a 4-D space; while x_{kj}, for $j=1, \ldots, 4$ are the four space-time coordinates of the k-th event, i.e. the three spatial ones plus the time reciprocal, all of them normalized with respect to their largest value; and

$$\bar{x}_i=m^{-1}\sum_{k=1}^{k=m}x_{ki},$$

for $i=1, \ldots, 4$ are the four coordinates of the centre of gravity of the foci.

Any quadric with its coefficients like the parameters above may be represented by a four-dimensional hyperellipsoid, the principal semi-axes of which are given, in length and orientation, by the principal parameters of the seismic sequence, i.e. by the eigenvalue square roots T_i $(i=1, \ldots, 4)$ of the matrix $[L]$, with $T_1>T_2>T_3>T_4$, and by its eigenvectors, u_{ij} $(i=1, \ldots, 4;$ $j=1, \ldots, 4)$.

Hypocentral determination errors, notoriously large especially relative to depth, average out in the construction of the matrix, causing a negligible effect on the centre of gravity determination, and even less on the eigenvector orientations. The lack of data for the smaller shocks of a sequence does not appear to influence the solutions (Ebblin and Michelini, 1986): tests carried out using only the largest ones yielded comparable results.

In 3-D, the effect of time was taken into consideration by sliding the window along the time sequence and analysing ellipsoids representing the same number of events established in advance while choosing the window length. The beginning of a new subsequence was defined by a change, greater than a chosen threshold, in the orientation of one ellipsoid with respect to that representing the preceding set of foci along the time sequence (Ebblin and Michelini, 1986).

However, together with a change in orientation of the ellipsoid, the onset of a new subsequence might be characterized not only by an anomalous jolt of the centre of gravity, but also by a marked change in the ellipsoid volume and/or flattening. Estimates of the dimensions and shapes of the individual hyperellipsoids, generally different one from another, can be obtained from the analysis of their invariants. Given the four invariants:

$$I_1=T_1+T_2+T_3+T_4$$

$$I_2=T_1\,T_2+T_2\,T_3+T_3\,T_4+T_1\,T_3+T_1\,T_4+T_2\,T_4$$

$$I_3=T_1\,T_2\,T_3+T_1\,T_2\,T_4+T_1\,T_3\,T_4+T_2\,T_3\,T_4$$

$$I_4=T_1\,T_2\,T_3\,T_4$$

then I_4, the last invariant and a product of the four semiaxes, is proportional to the hyperellipsoid's hypervolume, whereas the third and the second, proportional to the sums of the ellipsoidal and elliptical sections of the 4-D ellipsoids, may represent measures of their "hyperroundness". Hence, the parameter

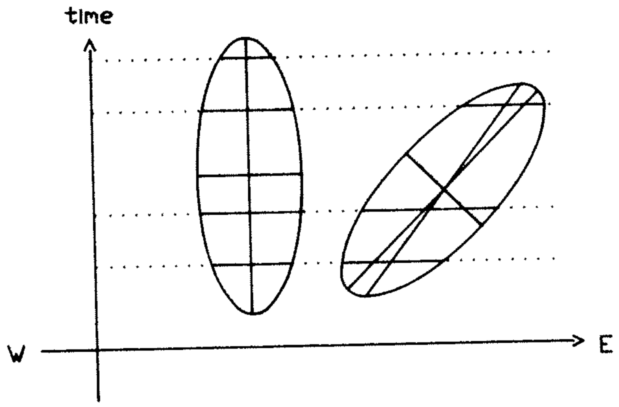

Fig. 1 — Two-dimensional longitude-time analysis: on the left, the ellipse principal major axis coincides with the line connecting the unidimensional baricentres; on the right, it is shown how this does not occur if the shocks propagate eastwards, and the corresponding ellipse forms an angle with the time reference axis.

$$R_4 = (I_2 * I_3) \;/\; (I_1 * I_4)$$

may be used to estimate the general degree of flattening normalized for the hypervolume of each hyperellipsoid, 6 being the value of hypersphericity up to infinity in the case of an infinitely flattened or elongated 4-D ellipsoid with respect to its hypervolume. In the present work, the nearly hyperspherical 4-D ellipsoids will not be considered in the results by applying a lower threshold to the R_4 values, on analogy with the works of Ebblin and Michelini (1986) and Michelini and Bolt (1986), otherwise, the principal axis orientations would be poorly defined.

If, furthermore, there is a high correlation between two of the spatial directions, it means that the foci are distributed on a plane, the orientation of which is given by the angle between the two directions. The moment matrix given by this set of foci is singular, and principal axis directions are again indeterminate. To avoid this, the rank of the matrix is reduced by a rotation by that angle.

The 3-D model proposed (Ebblin and Michelini, 1986) is based on the assumption that foci preferentially propagate along three orthogonal directions: the lateral edge of the active section of the fracture, the slip direction, and the normal to the fault, indicating the activation of other parallel ones. The distance reached in the three cases is different: the greatest for the first one, smaller for slip direction, and the smallest for the normal to the fault plane. Although this model, when compared with the fault plane solutions, appears to be a good representation of dip-slip focal mechanisms, it is conceivable that in strike-slip cases the intermediate and longest axes interchange.

Extension of the methodology to four dimensions adds a fourth principal axis characterizing the hyperellipsoid orientation in 4-D space. However, no pairs of the four orthogonal axes in 4-D space will necessarily be perpendicular when projected onto 3-D space; in fact, they will not be so unless the principal plane defined by them in 4-D space is normal to the time coordinate axis. The particular case of orthogonal triplets of projections occurs only when the fourth axis is parallel to the time coordinate and its projection onto 3-D space reduces to a point (Fig. 1, left).

Hence, in the 4-D representation, foci which propagate onto adjacent, subparallel discontinuities are not bound, as in 3-D, to a direction at right-angles to the fault which is defined by the two intermediate principal axes of the 4-D ellipsoid. Likewise, the slip direction need not necessarily be normal to the lateral edges of the fault set where stresses also concentrate. Moreover, although the projection direction of the fourth axis may be approximately deduced from that of the gravity centre displacements in the 3-D analysis, it is clear that they are not the same and that the latter results may occasionally lead to incorrect conclusions (Fig. 1, right).

The projection of the fourth axis has been interpreted as the seismic activation propagation

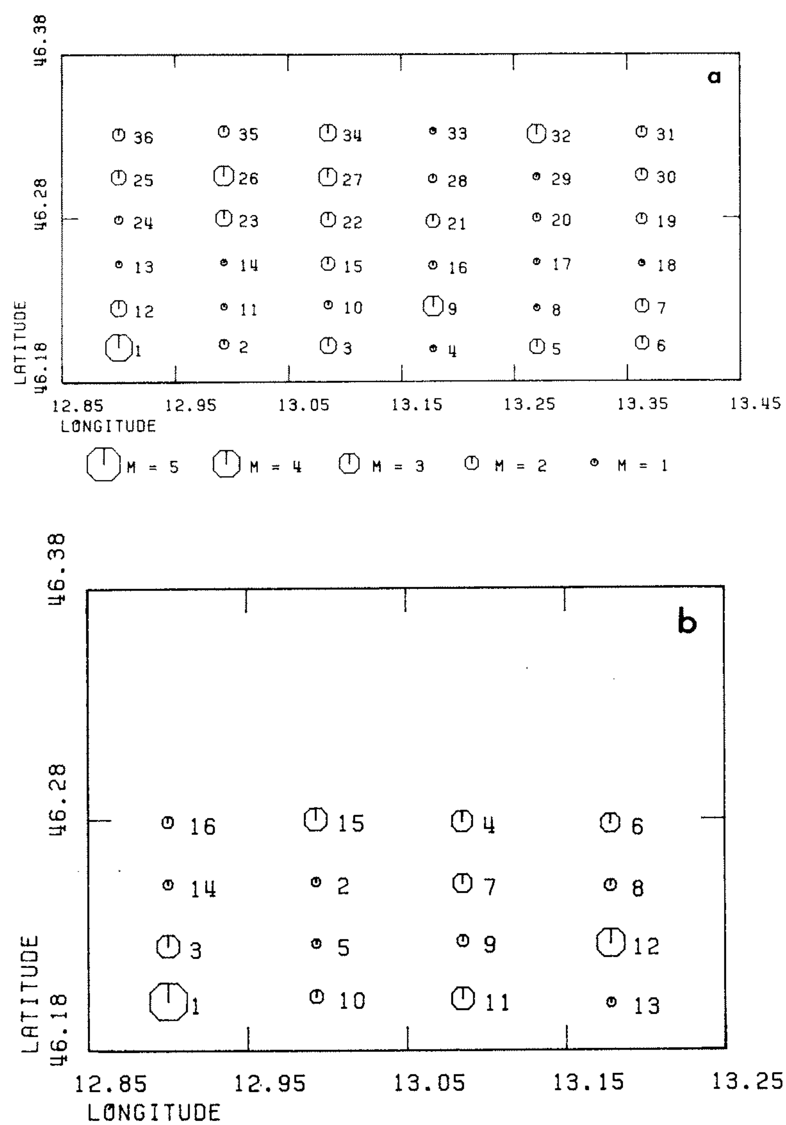

Fig. 2 — Spatial distribution a) of the synthetic epicentre set; b) of the second synthetic epicentre set. Temporal sequentiality is given by the number on the right.

direction. In order to test this hypothesis, a number of synthetic 3-D cases were reduced to 2-D. A set of 36 epicentres evenly distributed on a rectangular grid (Fig. 2a) were made to follow at similar time intervals, the first being in the SW corner, the next five moving progressively eastwards then, after a step northwards, westwards and so on to reach the last epicentre in the NW corner. Two of the three principal axes obtained in a N-E-time reference system projected onto the 2-D N-E plane turned out to be oriented N-S, and the third E-W (Fig. 3a).

On the contrary, if, in a similar grid, epicentres are made to follow preferentially in a ENE direction (Fig. 2b), the three principal axis projections are oriented N-S, E-W and ENE-WSW, thus suggesting that analogous relationships will hold in 4-D, and that the projection of the fourth axis may indeed represent the activation propagation direction, which does not necessarily coincide with the slip direction (Fig. 3b).

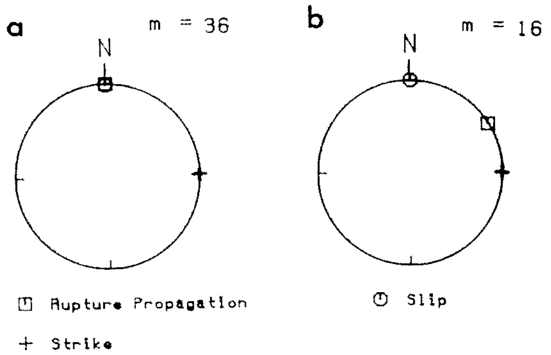

Fig. 3 — Stereogram (lower hemisphere) a) showing the projections of the time-space ellipsoid principal axes onto the E-W plane for the synthetic series of Fig. 2a; b) showing the projections of the time-space ellipsoid principal axes onto the E-W plane for the synthetic series of Fig. 2b.

AFTERSHOCK SEQUENCE SPACE-TIME ANALYSIS

The space-time characteristics of earthquake sequence have been studied very intensively for, among other things, any anomaly which might precede the occurrence of unusually large events. For example, a sudden increase in the number or frequency of earthquakes or of the sum of their energy is interpreted as a premonitory signal (Keilis-Borok et al., 1980a, 1980b).

The effect upon the 4-D ellipsoids of an event frequency increase, if not accompanied by a relative increase of the spatial concentration, is that of a hypervolume increase. On the other hand, if the frequency increase goes along with an increase in the spatial concentration, so that the corresponding hypervolume remains constant or even becomes smaller, then the anomaly will appear clearly in the flattening which is enhanced owing to the increased inhomogeneity in the space-time domain shock distribution. This provides an effective means of identifying swarms or bursts of seismicity.

On the contrary, a hypervolume decrease might reveal a marked spatial clustering of the hypocentres, which may be particular to some areas, perhaps even at the expence of an increased frequency. But, if instead, a gap or quiescence in the seismic activity marks the arrival of a larger earthquake, as evidenced by 2-D time-analyses (Kanamori, 1981; Mogi, 1985), this should be easily identified by scanning the sequence with the 4-D PPM since either a decrease of the hypervolumes or an increase of the flattening ought to be detected, followed by an inversion after the main shock.

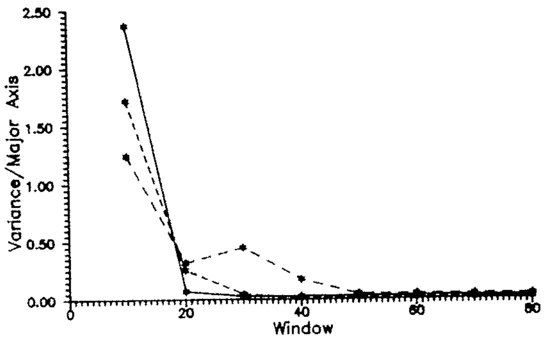

Fig. 4 — Window size (x-Axis) versus the average of the ratios between the mean distance of the hypocentres from the computed ellipsoid and its major axis: the results of the analysis of three different seismic series are shown.

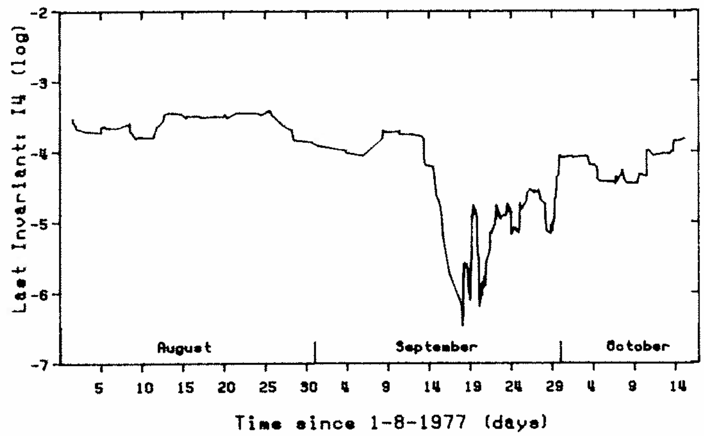

Fig. 5 — Time variation of the fourth invariant of the rupture hyperellipsoid proportional to its hypervolume. (1-8/14-10 1977, Friuli, north-eastern Italy).

Once the limits of each single sequence or subsequence are determined on the basis of the I_4 and R_4 variations, they may be extracted from the time continuum without fear of loosing meaningful details for the particular subsequence, or to the preceding and/or following subsequence, and a further separate analysis might be carried out on a homogeneous set of shocks, i.e. a set related to a single rupture process.

The proper window size is, among other things, dependent on the quality of the data. A criterion different to those of Ebblin and Michelini (1986), who considered ellipsoid flattening versus window size, and Michelini and Bolt (1986), who based the choice of the optimum width on the effect that larger windows had on eigenvector orientations, has been tried: the average of the ratios between the mean distance of the hypocentres from the computed ellipsoid and its major principal axis was compared to the window size (Fig. 4). Once more, the values obtained from different sequences dropped sharply while increasing the window size up to 20-30 but remained almost constant thereafter, suggesting, as in the other cases, that the best window size lies in this interval.

THE FRIULI NATURAL SEQUENCE

North-central Friuli (NE Italy), at the southern foot of the Eastern Alps, is marked by the merging of three tectonic systems: the E-W Alpine, the NW-SE Dinaric, and the NE-SW Venetian.

The fault plane solutions of the larger earthquakes of the first half of this century revealed a southerly maximum compression causing strike-slip on Dinaric discontinuities (Ebblin, 1976). But the destructive 1976 event, although still related to a N-S principal compression, was shown to be of a dip-slip nature (Mueller, 1977), whereas the focal mechanism of its aftershock sequence is far from evident (Ebblin, 1980).

The data on the shocks with magnitude greater than 1 obtained from the local seismometric network operated by the Osservatorio Geofisico Sperimentale, Trieste, starting from one year after the main shock (OGS, 1977), had been used to test the PPM in 3-D (Ebblin and Michelini, 1986). In particular, September 1977 is worthy of interest in that it is marked by comparatively higher seismic activity, comprising an $M_d=5.2$, and different focal mechanisms seemingly alternating in time (Ebblin and Michelini, 1986).

The same September 1977 Friuli data (OGS, 1977) are used here as an example of a natural aftershock sequence for elucidating the PPM in 4-D.

The sequence associated with and including the major shock of September 16th is marked by a sudden, strong hypervolume decrease (Fig. 5). The trend had returned to normal by the

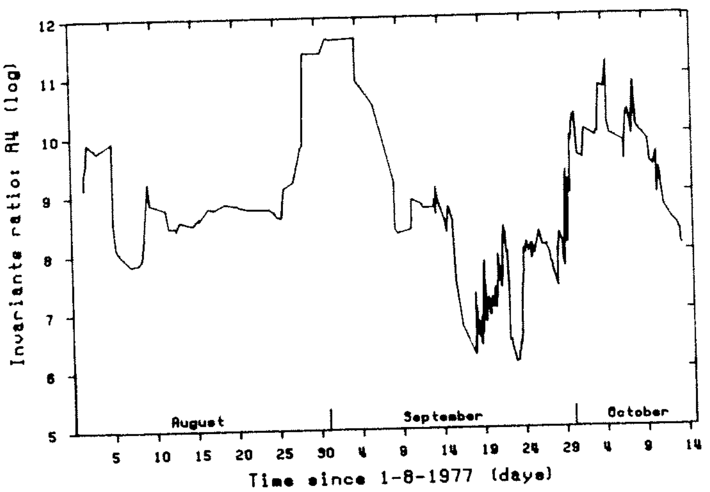

Fig. 6 — Time variation of the ratio R_4 among the invariants. (1-8/14-10 1977, Friuli, north-eastern Italy).

second half of October. On a logarithmic scale the recovery appears to be fairly linear with time, as might have been expected (Richter, 1958). The troughs coincide with the occurrence of major shocks on the 18th, 19th, 22nd and 28th, perhaps marking the beginning of different subsequences. Moreover, an additional minor phase of hypervolume decrease separated from the main one appears to forerun it.

The flattening parameter R_4 relative to the same sequence exhibits, instead, very different variations (Fig. 6) with a maximum at the beginning of September and a minimum on the 23rd.

However, an entirely different graph is obtained when R_4 is computed using hyperellipsoids "corrected" for the shape of the total sequence, represented by the 4-D ellipsoid of all the foci (Fig. 7, thin line).

Such a "correction" takes care of the fact that ellipsoids constructed with a set of points occurring at random in an ellipsoidal space are co-axial and display the same flattening as the space ellipsoid itself. The correction is performed by multiplying the matrices representing each ellipsoid by the inverse of that relative to the total sequence (Fig. 8) and is virtually equivalent to the "shape correction" by Lutz (1986). Hence such a "normalization" enhances the flattening and the elongation directions different from the general ones of the whole sequence.

Interestingly, R_4, relative to the normalized hyperellipsoids, shows (Fig. 7, thin line) a sharp anomaly in connection with the occurrence of the strongest September event (Fig. 5), and clear subpeaks marking the major aftershocks which followed.

Similarly, although the volume variations computed in 3-D are fairly similar to those in 4-D, the 3-D flattening parameter

$$R_3 = (I_1 * I_2) / I_3,$$

which may vary between 9 (spherical case) and infinity, displays changes which appear to be quite different and less abrupt than those of R_4 (Fig. 7, heavy line). In fact, there is no R_3 peak marking the occurrence of the larger shocks, but, instead, a slight increase might be detectable, forerunning the burst of activity, in agreement with other results from foreshock spatial analyses (Von Seggern et al., 1981).

HYPERELLIPSOIDS AND FAULTS

Fortunately, in the majority of the natural cases analysed, three of the four axis projections

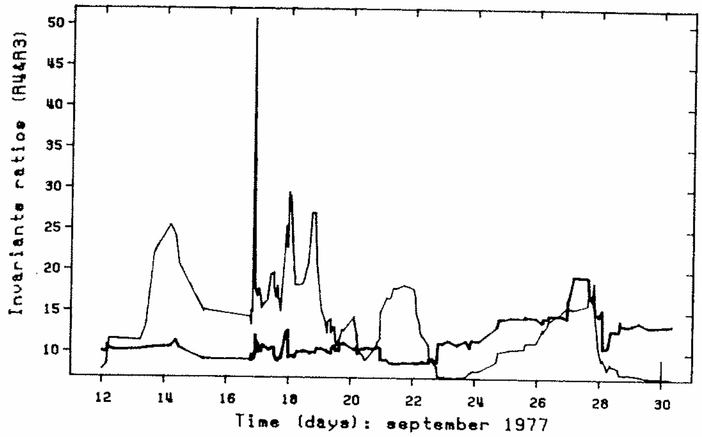

Fig. 7 — Time variation of the ratio R_4 among the invariants (thin line), and of the corresponding 3-D ratio, R_3 (heavy line), after shape correction (12-9/30-9 1977, Friuli, north-eastern Italy).

proved to be almost at right angles and subparallel to the corresponding axes obtained from the 3-D analysis, showing that the fourth one makes a small angle with the time reference axis.

All the directions referred to hereafter are those of the projections onto 3-D space of the four principal axes. Indeed, it does not seem very surprising that the propagation and the slip directions may not be parallel as long as they both lie in the fault plane.

Also, instances when the propagation direction lies in the plane determined by the shortest and intermediate axes, at an angle to both of them, may be easily interpreted as depicting cases of en echelon faulting.

It is more difficult to explain orientations of those propagations not closely aligned to any of the planes. In some of these cases, the propagation appeared to represent the migration of hypocentres from one cluster to another.

Indeed, the application of the Principal Parameters Method in three dimensions to September 1977 sequence (Ebblin and Michelini, 1986) and its interpretation corroborated the suspicion that the intricacy of the fault plane solutions was due to an E-W, Alpine set of overthrusts active together with NW-SE, Dinaric wrench faults. In other words, while the direction of maximum compression remained constantly oriented in a southerly direction, the intermediate and minimum principal axes were periodically reversing.

In agreement with the model above, the Alpine movements are tentatively associated with the local stress field and the Dinaric ones with the stress redistribution produced by the previous shocks.

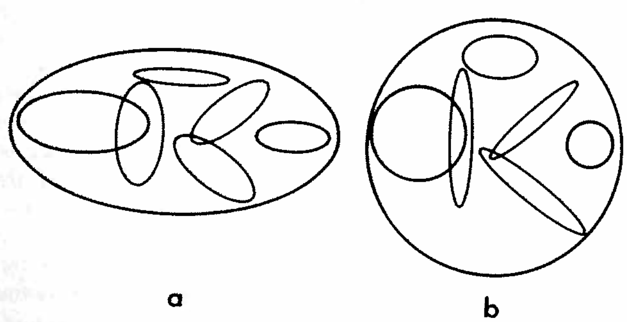

Fig. 8 — a) Distribution of variously oriented and shaped ellipses in an elliptical area. b) The same set after shape correction.

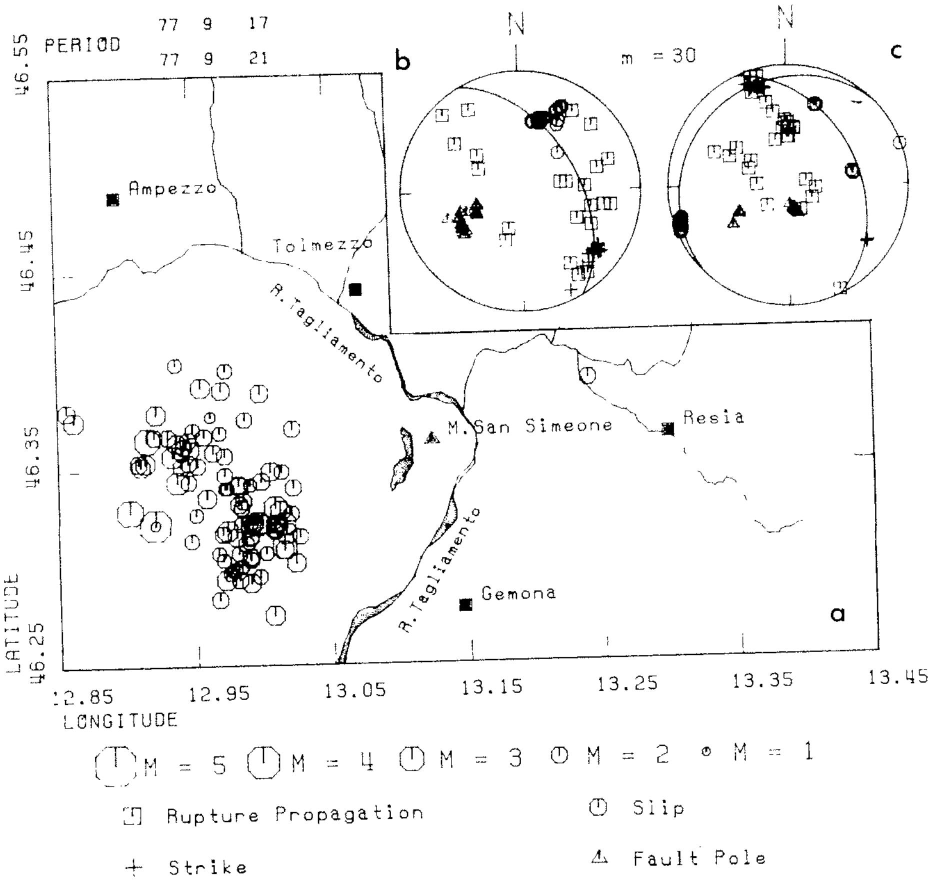

Fig. 9 — a) Spatial distribution of the Friuli (September 1977) sequence epicentres on a schematic map of the region (17-9/21-9 1977). b) Stereogram (lower hemisphere) showing the projections of the 4-D hyperellipsoid principal axes onto the 3-D spatial volume for the first part of the sequence. c) The same for the second part.

In an effort to discriminate between the times of Alpine and of Dinaric slip, it became apparent that an effective tool for establishing the onset of the different phases was necessary, and that extension of the method into a multi-dimensional space was a viable way to accomplish this.

The 4-D PPM appeared to yield useful results in that it permitted the definition of what are believed to be the beginning and end of the sequence, together with those of its relative subsequences, thus allowing a close-up, directional analysis of the three higher-order phases separately. The epicentral distribution relative to the first two subsequences (Fig. 9a) does not show any preferential alignment of points. On the contrary, the orientation of the principal axis projections (Fig. 9b) reveals the onset of a Dinaric fault, clearly slipping along its strike, which starts the sequence and propagates along its dip, followed by the activation of a sub-horizontal discontinuity (Fig. 9c) characterized by an Alpine NNW-plunging slip and a similar propagation direction which turns progressively vertical and perpendicular to the fault, thus suggesting that other, subparallel discontinuities were being activated. The last part of the seismic evolution may depict preferential trends in its epicentral distribution (Fig. 10a). And, indeed, they seem to be related to the PPM results which show that the faults were characterized by a quasi-cylindrical oscillation of their surface about an E-W, horizontal axis (Fig. 10b) with a final flip onto a new, subvertical fault. Dip-slip and strike propagation appear to be characteristic of these latter faults, which are stikingly similar in orientation to the nodal planes of the fault plane solution of the main May 6, 1976 shock.

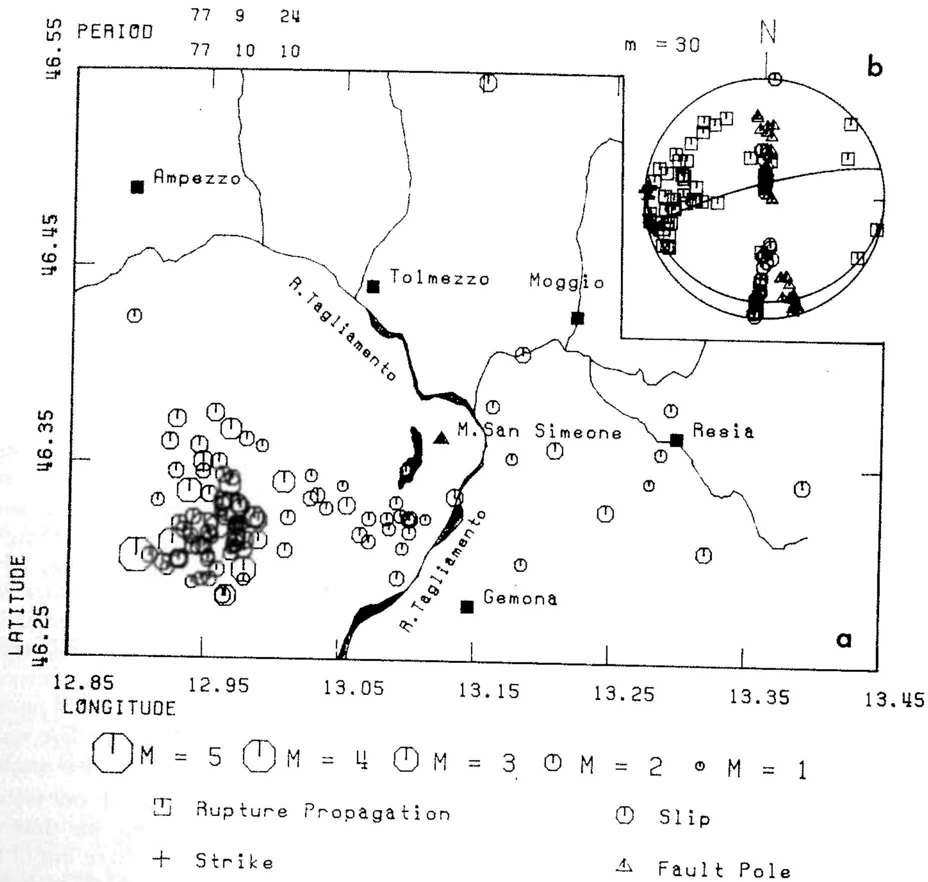

Fig. 10 — a) Spatial distribution of the Friuli (September 1977) sequence epicentres on a schematic map of the region (24-9/10-10 1977). b) Stereogram (lower hemisphere) showing the projections of the 4-D hyperellipsoid principal axes onto the 3-D spatial volume for the sequence.

DISCUSSION

The spatial distribution of earthquake foci is obviously related to the pattern and orientation of the seismogenic faults. Hence, when these are multiple, variably oriented, and/or of several orders, hypocentral clouds may reveal no anomalous concentrations, particular alignments, nor clear shape peculiarities. On the other hand, the image of hypocentral migration is alternatively connected with fracture propagation, progressive slip, or with the activation of other fractures, perhaps parallel and belonging to the same set. But irrespective of the particular physical cause guiding hypocentral propagation, from the continuum mechanics point of view, the shocks will not only occur preferably in places of low strength and shear stress concentration but also where additional stresses caused by previous quakes contribute. Thus the seismic process is envisaged as being characterized by a certain degree of causality. In general terms, each event is expected to produce a stress increase at the edge of the slipping surface, with possible maxima occurring in the slip direction and/or at right angles to it on adjacent discontinuities (Anderson, 1951; Price, 1966). These are the sites where the subsequent shock would occur (Mendoza and Hartzell, 1988) if both the stress field and the strength of the rocks were homogeneous. Therefore, vectors connecting foci to those immediately subsequent display preferential orientations; and although these do not necessarily coincide with the possible shape elongations of the cloud as a whole, they are believed to be meaningful indicators of the stress pattern and of the redistribution caused by the tremors.

A richer description of the shock propagation geometry comes from the PPM, which yields quadric equations that may be assumed to represent the so-called "rupture ellipsoids". The orientation of these ellipsoids, if interpreted in terms of the seismogenic fault planes and of the slip on them, has proved to be surprisingly consistent with the results obtained from fault plane solutions (Ebblin and Michelini, 1986; Michelini and Bolt, 1986).

A more complete analysis of the seismic process should further consider the distributions and correlations of the other two dimensions which with the spatial ones characterize an earthquake: time and energy. PPM takes into account time by sliding a window along the temporal sequence, thus following the time evolution of the rupture.

In this work, a four-dimensional approach to the seismic process has been made, by extension of the PPM to the four space-time dimensions: an additional fourth eigenvector is then available, which is related to the fracture propagation direction. Moreover, an examination of the hyperellipsoids' size and shape temporal variations provided an outline of the details of the hypocentral distributions which are meaningful for the partitioning of seismic catalogs into separate sequences and subsequences, and thus for discerning the time limits within which single fault sets are active, and for discretizing an otherwise continuous sequence into its homogeneous constituents. In fact, the onset of shock bursts is clearly evidenced by peaks representing the sudden spherical coalescence of hypocentres, irrespectively of the degree of flattening of the ellipsoid representing the total sequence, together with the even more striking departure from such flattening of the ellipsoids marking the beginning of the sequence. Both these variations appear to be more evident in a 4-D space-time analysis than in the corresponding 3-D spatial one, since the former offers a more vigorous geometrical, and to a certain extent physical, representation of the source volume evolution and its inferred relationship with the fault pattern. On the contrary, in the particular cases tested, the extension of the method to the five space-time-magnitude dimensions does not seem to add any significant information to the 4-D analysis.

The subsequent analysis of the principal axis orientations, carried out on isolated subsequences, takes into account the projections of the four axes down along the time axis onto 3-D space. Although not any longer necessarily orthogonal, most often, three out of four of the projections prove to be nearly co-axial with those of the 3-D analysis and thus showing a strong connection to the geometry of the fault. On the other hand, the fourth axis, which may or may not be parallel to any of the other three, depicts the fracture propagation direction, a relevant feature for the reconstruction of the fracture activation evolution.

In the particular example reproduced here, the method resolves the Friuli September 1977 seismic crisis into three parts which, when analysed separately, provide details about the rupture evolution. In fact it can be seen (Fig. 9) how an initial strike slip occurring on a Dinaric discontinuity first bends progressively to parallel its dip, then duplicates itself en echelon and further perpendicularly to the slipping part of the fault. Only later does the movement flip over onto a family of subhorizontal fractures (Fig. 10), slightly bent about an E-W axis, to end up on a subvertical plane conjugate to previous ones, extending progressively along its strike.

In conclusion, the geometry of the focal processes relative to an aftershock sequence may be effectively described by the Principal Parameters Method in 4-D. Such a straightforward technique allows one to find the sequence limits, together with those of its subsequences, and to infer the orientation of the seismogenic rupture, of its slip and propagation directions, and their evolution in time.

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