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FRACTAL DIMENSION TIME VARIATIONS IN THE FRIULI (NORTHEASTERN ITALY) SEISMIC AREA

Abstract. A fractal analysis of seismicity in the Friuli (northeastern Italy) seismic region is performed by applying the Correlation Integral Method to the hypocentres of the shocks registered by the local seismic network during the time interval 1977-1987. Two temporal windows of thirty days and of thirty events, respectively, are chosen to follow the long-term as well as the mid-term variations of the fractal dimension, and hence of the shock space distribution. A long-term component appears superimposed on an annual one and on the higher frequency terms. The beginning of a new seismic activity phase is marked by a long-period fractal dimension increase, which indicates the spreading of the hypocentres. A temporary mid-term fractal dimension decrease appears to preceed many of the greater energy events during the time period considered, indicating that, after the activation of the entire region, hypocentres tend to cluster and define the future rupture plane. Successively, a general fractal dimension decrease reflects the high clustering degree of the aftershock sequences and of the smaller ones of the post-seismic phases. The fractal dimension values obtained here lie in the interval 0.3-1.7, thus being consistently lower than the ones found with the same technique during a laboratory experiment by Hirata et al. (1987) and suggesting a lack of self-similarity in the seismogenic fracturing processes. This discrepancy may be due to either or both an uncompleteness of the seismic catalogue at the lower energies, and the tendency for the seismic activity in Friuli to define a surface rather than a volume, being limited to the uppermost 10-20 km of the crust.

INTRODUCTION

Part of the appeal of fractal concepts for describing natural phenomena is given by their ability to quantify with a single figure, namely the fractal dimension, qualities like roughness, fragmentation and irregularity, without referring to scale (Mandelbrot, 1977). Hence the property of self-similarity, i.e. the superposition of similar shapes of various dimensions, well-known to structural geologists, is inherent in the concept. One of the most striking examples of self-similarity is given by the minor folds commonly present along the geometry of major ones, reproducing in detail the latter. Another is the complex pattern of branching fractures which appears to be similar at all scales, thus suggesting that the same mechanism is responsible for their formation, acting identically on small scales as on larger ones (Tschalenko, 1970; Gay and Ortlepp, 1979; Segall and Pollard, 1980; King, 1983).

Seismicity and the associated fracturing processes have been recognized by many authors as belonging in the fractal framework (Kagan and Knopoff, 1980, 1981; Kagan, 1981a, 1981b, 1982; King, 1983; Sadovskii et al., 1984; Brown and Scholz, 1985; Turcotte, 1986; Scholz and Aviles, 1986; Aviles et al., 1987; Hirata et al., 1987; Okubo and Aki, 1987; Smalley et al., 1987; Yamashita and Knopoff, 1987; Hirata, 1989; Yamashita and Knopoff, 1989). The fractal dimensions were obtained by analyzing in turn the space and time distribution of

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Table - Fractal dimension of seismic process

Reference	D	Characteristic considered
Sadovskii et al., 1984	1.4-1.6	Epicentres distribution
Aviles et al., 1987	1.1-1.4	Fault length
Okubo et al., 1987	1.1-1.4	Fault length
Hirata, 1989	0.9-1.6	Fault distribution
Brown and Scholz, 1985	0.9-1.7	Rock surface
Hirata et al., 1987	2.25-2.75	Acoustic hypocentres distribution
Smalley et al., 1987	0.13-0.26	Shocks time distribution

the events, and the geometry of the fault pattern or their distribution, as summarized in the Table.

Finding the fractal dimension of the hypocentral distribution of a seismic sequence gives an estimate of the mode of propagation of the phenomenon. In fact, fractal dimension 1 describes, like that of the traditional Euclidean geometry, centres aligned along a straight line. A dimension 2 would depict shocks widely distributed on a planar surface and, similarly, a dimension 3 refers to centres filling up an entire volume. Any intermediate pattern may be described by a non-integer fractal dimension, the upper limit being the dimension field itself. Roughly, in 2-D, the fractal dimension is a measure of how many straight-line segments of fractional length connect the extremes of an unitary one (Fig.1). It is quite intuitive that the more the points constituting the vertices of the polygonals obtained scatter away from the straight line connecting the end points, the larger the fractal dimension. If such point distributions are self-similar, the same fractal dimensions are constant at whatever scale they are examined (Fig.2).

This work is intended to be an objective effort to perform a complete fractal analysis of the shock space distribution in a seismic region (Friuli, northeastern Italy). The distribution of the seismicity suggests that self-similarity is present on at least two orders of scale: smaller focal zones are recognizable in the larger, well-defined volume which constitutes the effective seismic area of Friuli. The various episodes of clustering, preceded and followed by the spreading of the shocks toward a new focal zones, are expected to influence the fractal dimension.

From among the various algorithms, the Correlation Integral Method (Hirata et al., 1987) was chosen for the Friuli data inversion. Such a choice allows the possibility to compare the results obtained here with those deduced using the same technique on the hypocentres of the acoustic emissions during a laboratory microfracturing experiment (Hirata et al., 1987), and thus to prove the self-similarity of the seismogenic fracturing processes.

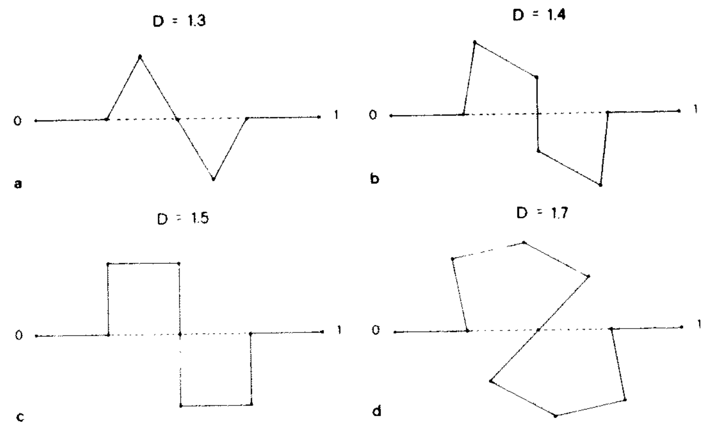

Fig. 1 - The extremes of a unit-length segment may be connected by a certain number of fractional-length r segments: in this case, the length r is 1/4. As the number N of segments increases (from a to d), the relative fractal dimension D increases also, since $D=\log N/\log(1/r)$.

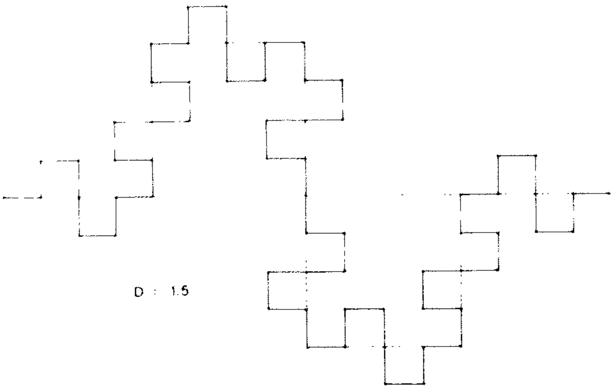

Fig. 2 - A self-similar polygonal presents the same fractal dimension, D = 1.5, even when two or more hierarchy orders are present.

AN EXAMPLE OF FRACTAL ANALYSIS

The size distribution of each characteristic of a fractal object is given by a power law, the exponent being the fractal dimension (Mandelbrot, 1977). The space distribution of objects like earthquakes may be analysed using the Box-Counting algorithm (Sadovskii et al., 1984). An active region limited by a square of side length R is divided into (R/r^2) square boxes of side length r. Let N(r) be the total number of boxes of any given size containinig one or more shocks. If the distribution is self-similar, then the number N(r) increases with the areal decrease of the single boxes and is a function of r^{D}, D being the fractal dimension. A limit to the applicability of the Box-Counting algorithm, when the number of data is small, is the instability of the fractal dimension obtained, which is connected with the choice of the location and size of the boxes. A different method, the Correlation Integral Method, gives the fractal dimension for small data sets. The procedure is very similar to the one above, but instead of square box areas containing events, the distances between the hypocentres are considered, so that the resulting measure is independent of the width and shape of the region. The number of pairs of shocks at distances smaller than a certain length r, progressively reduced, is plotted on a bilogarithmical diagram versus the length itself; the slope of the straight line again gives the fractal dimension, or better, following Grassberger (1983), its lower limit (Hirata et al., 1987).

The data used in the present analysis are those obtained from the local seismological network operated by the Osservatorio Geofisico Sperimentale, Trieste, over the seismic area of Friuli, northeastern Italy (OGS, 1977-1981, 1982-1987). Since the seismic activity of Friuli is weak compared with other regions in the world, the Correlation Integral Method is adopted. From prior inspection, time variations of the dimension were expected; thus, a temporal window of thirty days was used to scan the catalog starting from the time of operation of the local network in 1977 (Fig.3). Although the values of the fractal dimension using this procedure are averaged, some interesting features are apparent. There are three components in the variations: a high-frequency term, an annual one, and a long-period component, the latter two being more evident after the highest frequency energies are removed using a low-pass filter (Fig. 3b). The spectral analysis of the series confirms the presence of a strong long-period component, of about three and a half years, and of an annual one over which a six month and higher-frequency terms are superimposed. Regarding the long-period term, a general decrease is detected after the larger shocks of September 1977, generally considered aftershocks of the May 6, 1976 event. The minimum is reached in 1980, while a recovery begins in 1981. A further decrease follows the event of February 10, 1983 while a new slow increase seems to characterize the end of 1987. A comparison with the results obtained using the Box-Counting algorithm was also done: although, in disagreement with the theory (Grassberger, 1983), the values of the fractal dimension are lower in the latter case, but the trend is almost the same. This compa-

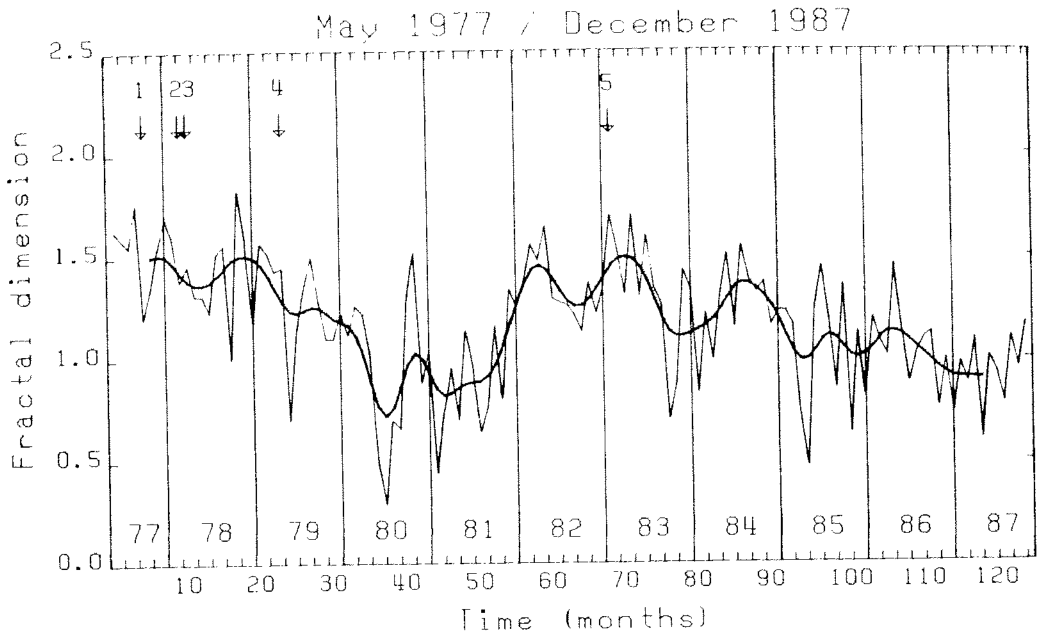

Fig. 3 - Fractal dimension variations over a ten year interval, obtained by scanning the local network (1977-1981: Friuli Venezia-Giulia network; 1982-1987: North-Eastern Italy network) with a temporal window of thirty days (thin line). A low pass filter is applied in order to remove the highest frequency energies (heavy line). The greater energy earthquakes of the interval are represented by an arrow and a progressive number. 1: September 16, 1977, ML=5.2; 2: February 20, 1978, ML=4.2; 3: April 3, 1978, ML=4.2; 4: April 18, 1979, ML=4.8; 5: February 10, ML=4.1.

rison between the two methods was done using the epicentres, rather than hypocentres, of the shocks.

The presence of a strong annual component in the fractal dimension variations is explained by the seasonal oscillation that characterizes the seismic activity of Friuli. No apparent correlation appears, on the other hand, between the highest frequency variations and the seismicity.

A second, more detailed analysis was performed, focusing on the intervals immediately preceding and following the strongest shocks in Friuli, by using a temporal window of thirty events (Figs. 4 to 7).

The fractal dimension values so obtained oscillate between 0.3 and 1.7, the minimum being registered before the event of April 18, 1979 ML=4.8 (Fig. 6), and the maximum during the aftershock sequence following the main shock of September 16, 1977, ML=5.2 (Fig. 4). The fractal dimension oscillations reflect, then, the shock clustering variations, being the first months of 1979 characterized by high clustering, with three principal focal zones where the activity concentrate, while after the September 16, 1977 main shock, the seismic activity propagates from the hypocentral zone eastwards, tending to involve the whole seismic area of Friuli (Rossi and Ebblin, 1990). A fractal dimension decrease, and hence clustering, seems to precede both the 1977 main event and the February 10, 1983, ML=4.1 shock (Fig. 7). A new decrease follows the major aftershock of the 1977 earthquake of September 28, ML=4.8, while, after a temporary decrease, the fractal dimension tends to increase after the 1983 event. The pattern of the variations during 1978 (Fig. 5) and 1979 (Fig.6) appears more articulate, the first being characterised by the occurrence of two shocks with ML=4.2 on February 20, and April 3 respectively, and the second by a series of shocks of magnitudes between 3 and 4.8, which occurred between March and August. With regard to 1978, strong fractal dimension decreases precede the first event and follow the second, suggesting a possible correlation of the two earthquakes, while in 1979 the fractal dimension tends to decrease continuously from March to August, in spite of its many short-time variations.

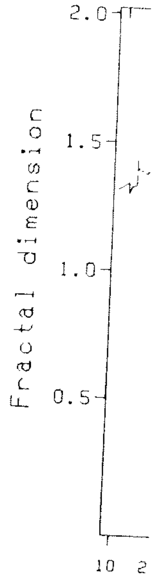

Fig. 4 - Fractal
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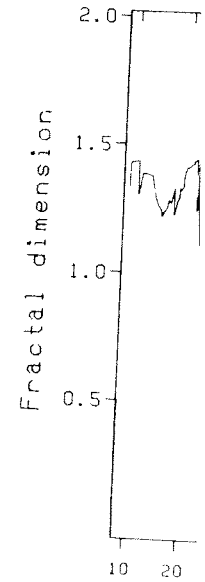

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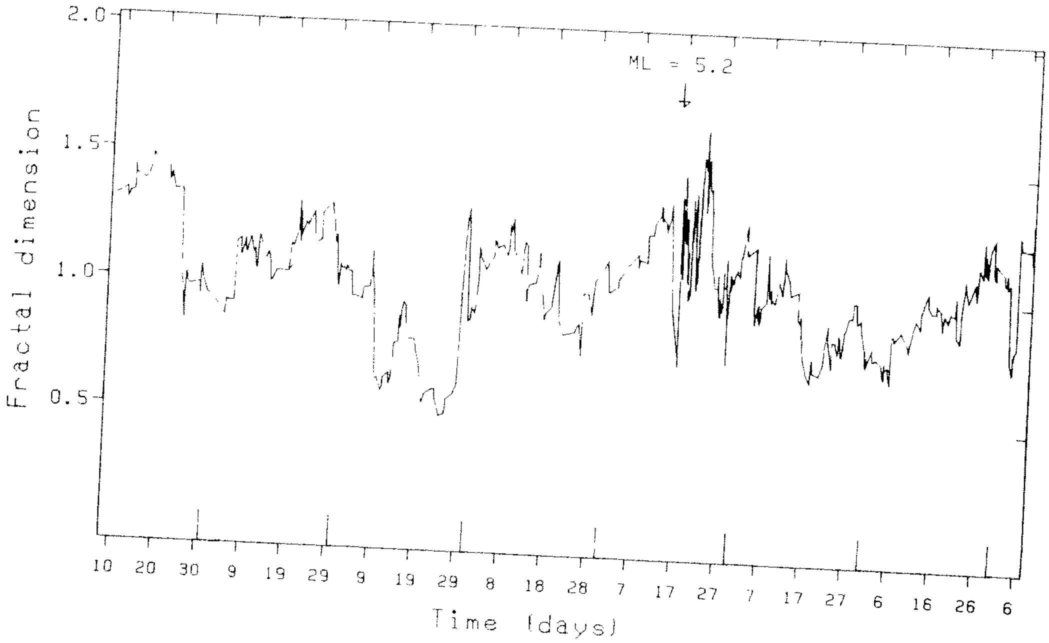

Fig. 4 - Fractal dimension time variation in the interval May 9, 1977 - December 7, 1977. An arrow marks the main event of September 16, 1977.

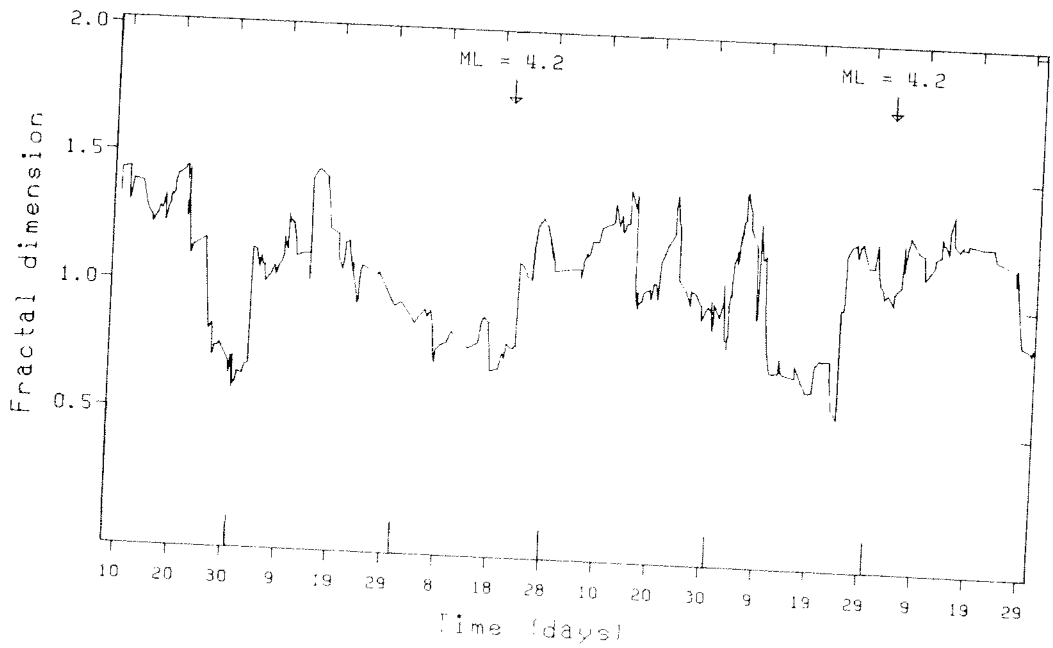

Fig. 5 - Fractal dimension time variation in the interval December 9, 1977 - May 31, 1978. Two arrows mark the main events of February 20, 1978 and April 3, 1978.

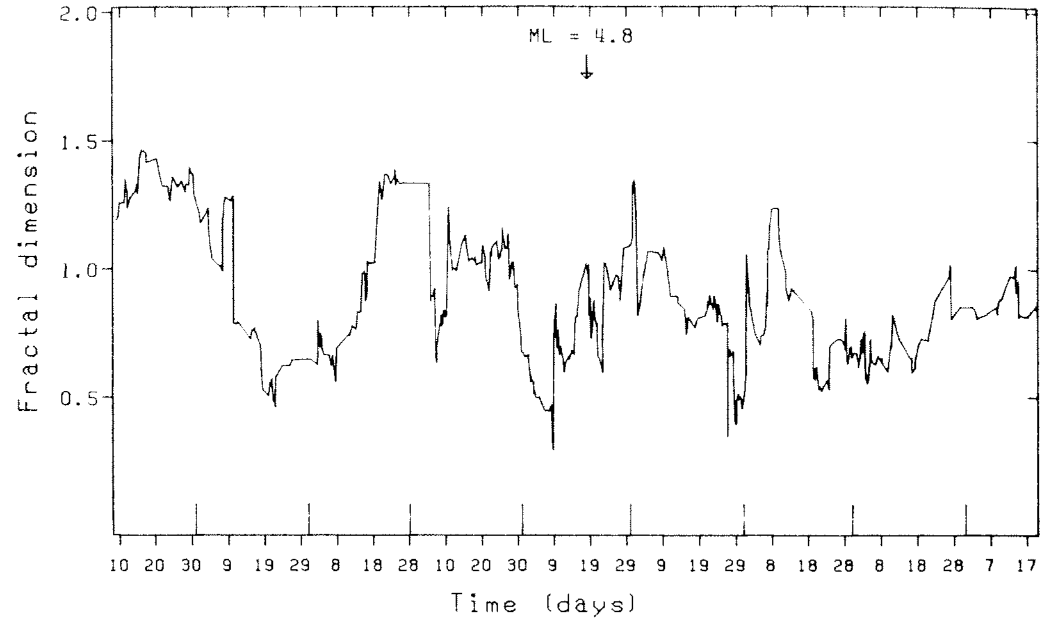

Fig. 6 - Fractal dimension time variation in the interval December 9, 1978 - August 19, 1979. An arrow marks the main event of April 18, 1979.

DISCUSSION

Synthetic models, based on fractal concepts, seem to approximate seismic series or fault patterns quite well (Kagan and Knopoff, 1980, 1981; Kagan, 1981a, 1981b, 1982; King, 1983; Turcotte, 1986; Yamashita and Knopoff, 1987, 1989), however, the inverse problem solutions are more puzzling. In fact, the fractal dimension of natural seismic sequences or fault patterns does not appear to be constant at all scales (Smalley et al., 1987; Aviles et al., 1987), thus suggesting, as in the case of other natural phenomena, the need of a proper multifractal-functions description (Mandelbrot, 1989).

This work presents a fractal analysis of the space distribution of the hypocentres of the shocks in Friuli (northeastern Italy) over a ten-year time interval. The long-term fractal dimension variations over the whole time interval were calculated by scanning the seismic catalog of the local seismic network of Friuli using two temporal windows of different width. Whereas the first, of thirty days, is independent of the shock number, the second, of thirty events, follows more accurately the space distribution time variations in the months immediately preceding and following the greatest shocks.

The results of both kinds of analysis show how fractal dimension may be a practical tool for quantifying and describing the shock distribution variation in a region. The long-period variations of the fractal dimension evidence the various phases which preceed and follow the main events in the region during the time interval considered.

The beginning of a new seismic activity phase is mainly marked by a long-period fractal dimension values increase, which indicates that seismic activity tends to scatter throughout the area. After the occurrence of a greater energy event, there is on the contrary a general fractal dimension decrease, due to the high clustering level of the aftershock sequences and of the other small sequences activated in the post-seismic phase. It is noteworthy that the same periods during which a fractal dimension inversion is observed are characterized by an inversion in the strain rate too, as results from tilt and strain measurements performed in the Friuli seismic area (Mao et al., 1989, 1990).

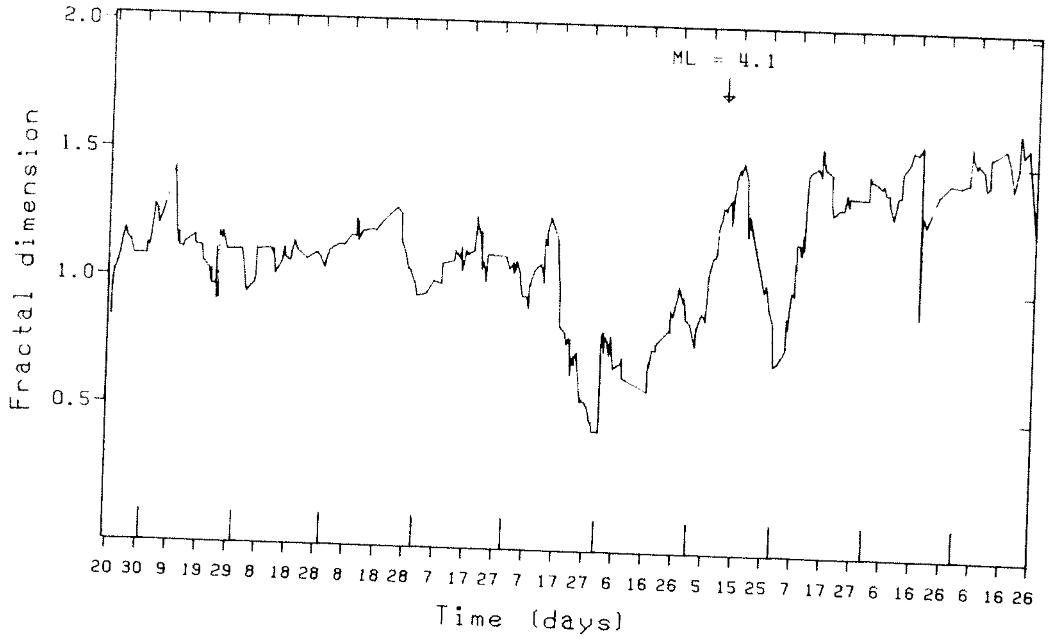

Fig. 7 - Fractal dimension time variation in the interval July 20, 1982 - May 25, 1983. An arrow marks the main event of February 10, 1983.

On the other hand, from inspection of the higher frequency variations obtained from the principal seismic sequences, a fractal dimension decrease appears to precede both the 1977 and 1983 main events and the first 1978 shock. The occurrence of a large number of small sequences during the spring months of 1979 causes a gradual decrease of the fractal dimension.

This behaviour is in agreement with the observations of Scholz (1968), Mogi (1985) and Hirata et al. (1987) on laboratory experimental data: after an initial activation of the entire region, as stress increases, leading to fracture, shocks begin to cluster and define the rupture plane (Fig. 8), according to a fractal dimension decrease. With such premises, the 1977 and 1983 events may be thought of as the end of the fracturing process, which started when shocks began to cluster. Analogously, the two 1978 shocks and the whole 1979 activity may be considered expressions of the same phenomenon, which caused the activation, in turn, of different zones located at the border of the Friuli principal seismic area.

It would thus appear that the Friuli seismic activity is characterized by a high percentage of causality. Here the occurrence of a major earthquake, like the 1976 and 1977 ones, causes a stress redistribution that involves the whole region, producing the successive activation of other seismogenic zones at the border of the rupture, as in the case of the 1978 and 1979 activity. After a subsequent period of seismic quiescence, as in 1980-1981, a new phase of increasing stress begins, with low-energy seismicity which progressively affects the whole region. A few months before a main shock, the occurrence of small sequences indicates the starting of the fracturing process, according to dilatancy theories (Scholz et al., 1973; Stuart, 1974) or the barrier model (Das and Aki, 1977; Aki, 1979) or the asperity model (Kanamori, 1981). Its end may be considered to be marked by the occurrence of the main shock, as observed for the 1983 event. It is noteworthy that a similar behaviour, a seismic quiescence followed by an increased activity just before the main event, which is common to many shallow events in different seismic regions, is also shown by the synthetic series generated in the random stress model of Von Seggern (1982), based on the well-known blocks-and-springs one (Burridge and Knopoff, 1967; Dieterich, 1972). Such a behaviour is in disagreement with a strictly self-similar fractal model of shock occurrence, according to which a smaller cluster is supposed to repeat the same shock distribution of the whole seismic region and hence, keeping the same fractal

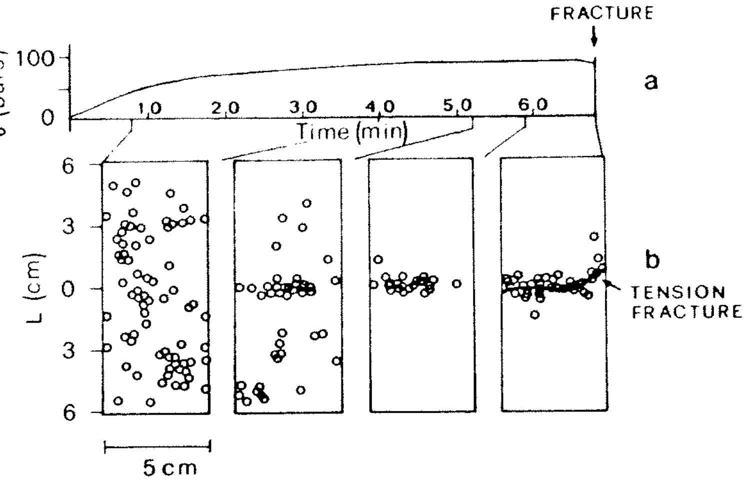

Fig. 8 - Variation of epicenter clustering when a granite sample is bent and fractured. a): Temporal change in stress; b): Two dimensional distribution in each period and the location of the final rupture plane (from Mogi, 1985, redrawn).

dimension D. It would thus appear that the Friuli space shock distribution is not self-similar in time, although the discrepancy in the fractal dimensions may be due to the incompleteness of the seismic catalogue at the lowest energies.

The fractal dimensions of the Friuli 1977-1987 seismic activity lie in the interval $0.3 \leq D \leq 1.7$.

Values even smaller were inferred during the systematic analysis of this ten year period, for the years 1980 and 1981, when the seismic activity was very low indeed and highly clustered in space. A fractal dimension lower than 1 in 3-D may be easily explained by a very high clustering degree, as verified during 1979, and perhaps also, as in the 1980-1981 case, with a period of seismic quiescence. The greatest values were registered immediately after the September 1977 and 1983 main shocks. Generally, they correspond to a propagation of seismic activity from one focal zone to another, or often several, and hence to a shock distribution over the whole region.

Comparisons between different values of fractal dimension is possible only if the characteristics examined and the dimension field are the same. In fact, the fractal dimension depends on the dimension over which the analysis is performed: the fractal dimension of a surface in 3-D differs from that relative to the projection of it onto a planar section of a 3-D volume. It follows that the fractal dimension values obtained in a certain dimension may be not extended simply to the higher or lower one by adding or subtracting a unit, as is often done (Andrews, 1980; Von Seggern, 1981; Hirata et al., 1987).

The present results have been compared with those of Hirata et al., (1987) obtained from the fractal analysis of the space distribution of the hypocentres of the acoustic emissions during a constant stress experiment. The latter are significatively larger, lying in the interval 2.25-2.75.

Such a difference suggests that the seismogenic fracturing process is not self-similar at every scale, as already deduced from a fault length analysis (Aviles et al., 1987), or from a rock surfaces study (Brown and Scholz, 1985). Large differences of fractal dimension may be due, however, to infinitesimal perturbations of the variables involved. Any lack of homogeneity in the regional coverage of the seismic network, resulting in an uncompleteness of the seismic catalog at the lower energies, may cause a great difference in the fractal dimension, in comparison with that relative to a laboratory experiment. But the depth location of the shocks may play a far greater role in controlling fractal dimension values, not only due to the errors in its determination, but also to the fact that the seismicity in Friuli is very shallow being limited to the uppermost 20 km of the crust, and generally concentrated in the uppermost 10 km. Fractal dimension values not greater than 2 are then justified by the tendency for the seismic activity to define a surface, rather than a volume, while the small size of the laboratory sample may lead to the hypocentres occurring in every direction.

On the other hand, the great instability of the inversion results suggests great caution in its use, and the necessity to follow proper criteria in the choice of the data sets to analyze: a systematic scanning of the seismic catalog was adopted here, using temporal windows of appropriate width, in order to evaluate possible variations with no fear of loss of information. In spite of these limitations, the fractal dimension may be a useful measure for representing the clustering degree of seismic events in a region, its evolution in time through a long-term analysis like the one proposed here, and for detecting all anomalies that may be related to the occurrence of a large shock, focusing on some particular portions of the seismic catalogue.

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